of current research. The reviewer was particularly happy to see the inclusion of the next topic: plane cubic curves. This is dealt with in a long chapter, culminating in a classification of the singular and nonsingular cubics in $PG(2, q)$. There follow interesting chapters on plane $(k\colon n)$-arcs and blocking sets, with a final chapter of detailed analysis of $PG(2, q)$ for small values of $q$.

One very helpful feature of the format of the text is a concluding section of each chapter, entitled "notes and references". Here the credits for the work of each preceding section are given, thus freeing the main text of a plethora of individual references, which would hamper the smooth flow of the development. The book is essentially self-contained and serves well the dual purpose of a good reference source for the expert and a vehicle of self-study for the newcomer to a fascinating but relatively unknown subject.

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*Translation planes*, by Heinz Lüneburg, Springer-Verlag, Berlin and New York, 1980, ix + 278 pp., $29.80.

As the author says in his foreword, infinite translation planes are not completely disregarded but finite ones form the main theme of the book. Finite projective and affine planes have interest for mathematicians in many other areas. First, they are important for combinatorialists, including people in design of experiments and in coding theory. Many combinatorial structures have natural representations associated with finite projective planes. Projective planes (finite or infinite) have obvious significance for the foundations of geometry. If one were to look for the field of interest whose adherents were most likely to have a strong side interest in projective planes a strong candidate would be the field of finite groups. Many of the early developments in projective planes came from people working on nonassociative division rings (semifields).

The known finite projective (or affine) planes are closely related to translation planes.

One way of looking at the Euclidean plane is to identify the points with elements $(x, y)$ of a two-dimensional vector space over the reals. The lines through the origin are one-dimensional vector spaces; the other lines are translates of the lines through the origin. This book develops the following analogous way of representing a translation plane: Let $V$ be a vector space which is a direct sum of two copies of a subspace $V_1$ so that $V = V_1 \oplus V_1$. Each element of $V$ can then be represented by an ordered pair $(X, Y)$, $X$, $Y \in V_1$. If $V_1$ has dimension $r$ then $V$ has dimension $2r$ and the sets of vectors for which $X = 0$ or for which $Y = 0$ are $r$-dimensional subspaces isomorphic to $V_1$. A *spread* defined on $V$ is a class of $r$-dimensional subspaces (the *components* of the spread) such that each nonzero element belongs to exactly one component and $V$ is the direct sum of each pair of distinct components. Without loss of generality $\{(X, Y)|X = 0\}$ and $\{(X, Y)|Y = 0\}$ are components and each other component has the form $\{(X, Y)|Y = XM\}$ where the mapping $X \to XM$ is a nonsingular linear transformation on $V_1$.

The points of the translation plane are elements of $V$ and the lines are the components of the spread and their translates. The same vector space $V$ may admit different spreads. Translation planes arising from different spreads can be isomorphic but they need not be so except when $r = 1$ or the number of elements of $V_1$ is less than 9.

One can then convert $V_1$ into a "quasifield". A quasifield roughly resembles a division ring except that multiplication need not be associative and that the distributive law is only required for multiplication on one side–i.e. $(a + b)c = ac + bc$ but $c(a + b)$ is not necessarily $ca + cb$. Much of the early interest in translation planes was directed primarily at the quasifield, especially in the cases where the full distributive law holds. Present investigators are more likely to concentrate on the representation already outlined.

If $(a, b)$ is a fixed vector, the mapping $(X, Y) \to (X + a, Y + b)$ is a *translation*. The translations form an abelian group of collineations sharply transitive on the points. The collineations fixing the zero vector constitute the *translation complement*. The translation complement is the subgroup of the nonsingular semilinear transformations on the vector space which preserves the spread. The present trend is to concentrate on characterizing translation planes by the nature and action of the collineation group.

The generalized André planes were initially constructed by modifying field multiplication to obtain a quasifield. They were characterized by Lüneburg in terms of an abelian subgroup of the translation complement which fixes two components of the spread and has the additional property that, for each component, the stabilizer of the component acts irreducibly.

Wagner [**18**] has shown that a finite affine plane admitting a collineation group transitive on lines must be a translation plane. Kallaher [**6**] and Liebler [**9**] have shown that a finite affine plane must be a translation plane if it admits a collineation group which acts as a rank 3 permutation group on the affine points. In this case the translation complement has exactly two orbits in addition to the zero vector which, of course, is in an orbit by itself. If one of these two orbits is the union of the nonzero vectors on two components of the spread the plane must be a generalized André plane except for a few cases related to the irregular nearfield planes. Some of these exceptional cases are due to Lüneburg. (The reviewer feels that Foulser's paper on *Group replaceable nets* [**2**] throws some light on this construction.) Thus, we have a solution to one part of the general problem: "Which finite translation planes are rank three planes?" Kallaher [**7**] and Liebler [**9**] have survey papers on this question. Kallaher and the reviewer [**8**] have a paper published recently in which the question is answered when the dimension of a component of the spread is odd. (The planes are semifield planes or generalized André planes.)

So far, we have given a very brief summary of some of the topics covered in the first three chapters.

The reader would be disappointed if a book by Lüneburg on translation planes did not include a chapter on the geometry of the Suzuki groups. There is indeed such a chapter.

There also is a chapter on the results of Hering **[4]** (based on earlier work of the reviewer **[11]**) on subgroups of the translation complement generated by affine elations (shears).

The final chapter deals with planes of order $q^2$ admitting $SL(2, q)$. The underlying vector space has dimension 4 over $GF(q)$ and can be interpreted as a three-dimensional projective space. The planes of Hering **[3]** and Ott **[16]** come from a four-dimensional irreducible representation of $SL(2, q)$ and are intimately related to the twisted cubics.

The reviewer feels that the whole subject of finite translation planes is becoming more and more a matter of group representation. It is not a mere matter of giving characters for group elements but of obtaining explicit representations which act on translation planes. One starts with a vector space of given dimension over a given field, asks which groups isomorphic to subgroups of the general linear group can be represented on translation planes defined by spreads on the vector space. One strong restriction is that the dimension of the subspace pointwise fixed by any element cannot exceed the dimension of a component of the spread.

The author says that the book presupposes considerable knowledge of projective planes and algebra, especially group theory. Actually it could be used as a textbook for a student with quite a bit less background than this suggests if the instructor were in a position to do a small bit of supplementation and to suggest a few sections to skip. Undergraduate courses in classical projective geometry and modern algebra would probably suffice under these conditions. Hughes and Piper **[5]** or Pickert **[17]** would be more than adequate as would Dembowski **[1]**. Actually the beginner might do well to read this book before reading Dembowski.

The book does not pretend to cover everything known about finite translation planes. To quote: " . . . I selected mainly those topics I had touched upon in my own investigations". Thus it is not intended to be a reference book. Actually it will serve this purpose very well for those topics which are included. The proofs are often more carefully carried through than they were in the original papers.

The reviewer has been unable to exercise enough self-restraint to keep from mentioning that he has written several survey articles on finite translation planes **[12]**–**[15]**. Even the "Lecture Notes" do not go into the detail that this book does. However one or more of these might serve as a useful supplement since they do cover some ground not mentioned in the book under review. The chief value of the report to the Pullman Conference **[13]** is that it contains the most nearly up-to-date list in print of known finite translation planes.

Our list of references is not intended to be complete. We have referred to some, but not all, of the papers underlying the material in the book. We have not, for instance, included references to any of Lüneburg's papers. The following references give supplementary material: **[1]**, **[2]**, **[5]**, **[7]**, **[8]**, **[10]**, **[12]**–**[15]**, **[17]**.

## REFERENCES

1. P. Dembowski, *Finite geometries*, Springer-Verlag, Berlin and New York, 1968.

2. D. Foulser, *Replaceable translation nets*, Proc. London Math. Soc. **22** (1971), 235–264.

3. C. Hering, *A new class of quasifields*, Math. Z. **118** (1970), 56–57.

4. ______, *On shears of translation planes*, Abh. Math. Sem. Ham. **37** (1972), 258–268.

5. D. R. Hughes and F. C. Piper, *Projective planes*, Springer-Verlag, Berlin and New York, 1973.

6. M. J. Kallaher, *On finite affine planes of rank* 3, J. Algebra **13** (1969), 544–553.

7. ______, *A survey of weak rank* 2 *affine planes*, Proc. Internat. Conf. on Projecture Planes, (M. J. Kallaher and T. G. Ostrom (eds.)), Wash. St. Univ. Press, Pullman, Washington, 1973.

8. M. J. Kallaher and T. G. Ostrom, *Collineation groups irreducible on the components of a translation plane*, Geom. Dedicata **9** (1980), 153–194.

9. R. A. Liebler, *Finite affine planes of rank three are translation planes*, Math. Z. **116** (1970), 89–93.

10. ______, *On finite line transitive affine planes of rank* 3, Proc. Internat. Conf. on projective Planes, Wash. St. Univ. Press, Pullman, Washington, 1973.

11. T. G. Ostrom, *Linear transformations and collineations of translation planes*, J. Algebra **14** (1970), 405–416.

12. ______, *Finite translation planes*, Lecture Notes in Math., vol. 158, Springer-Verlag, Berlin and New York, 1970.

13. ______, *Classification of finite translation planes*, Proc. Internat. Conf. on Projective Planes, Wash. St. Univ. Press, Pullman, Washington, 1970.

14. ______, *Recent advances in finite translation planes*, Foundations of Geometry, (P. Scherk (ed.)), Univ. Toronto Press, Toronto, Canada, 1976.

15. ______, *Finite translation planes, an exposition*, Aequationes Math. **15** (1977), 121–133.

16. U. Ott, *Über eine neue Klasse endlicher Translationsebene*, Math. Z. **143** (1975), 181–185.

17. G. Pickert, *Projektive Ebenen*, Springer-Verlag, Berlin and New York, 1955.

18. A. Wagner, *On finite affine line transitive planes*, Math. Z. **87** (1965), 1–11.

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*Fermat's last theorem, a genetic introduction to algebraic number theory*, by Harold M. Edwards, Graduate Texts in Math. Springer-Verlag, Berlin and New York, 1977, xv + 410 pp.

13 *Lectures on Fermat's last theorem*, by Paulo Ribenboim, Springer-Verlag, Berlin and New York, 1979, xvi + 302 pp.

For more than three centuries many good and many not so good mathematicians have attempted to prove Fermat's last theorem. While the collected efforts of these mathematicians have not yet led to a solution of this problem, much is now known about the problem and more importantly much new mathematics has been discovered in the process of working on the conjecture.

Fermat's last theorem can be simply stated as: Show that $x^n + y^n = z^n$ has no integral solutions with $n > 2$ and $xyz \neq 0$. It clearly sufficies to prove this result for $n = 4$ and $n = p$, an odd prime. When $n = p$, the theorem has been traditionally separated into two parts called case 1 and case 2. The first case is to show the equation has no solution with $xyz \not\equiv 0 \pmod p$ and the second is to show no solution exists with $xyz \equiv 0 \pmod p$.