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Singular homology theory, by William S. Massey, Graduate Texts in Math., Springer-Verlag, 1980, xii + 265 pp., $24.80.

Mathematics students normally encounter a mixture of courses on topics in pure mathematics, such as number theory or modern algebra, and courses in which mathematical techniques are applied to solve problems in the physical and biological sciences and engineering. But often the first occasion offering a student an opportunity to apply techniques from one branch of mathematics to solve theoretical problems in another is an introductory course in algebraic topology. Even for the student whose primary interest lies in another field, the subtle strength of these methods can be a source of genuine excitement and fascination. One need only spend some time trying to prove Brouwer's Theorem directly, that a continuous map of the closed n-disk D^n to itself must have a fixed point, to appreciate the effectiveness of homology theory.

These refined tools are not easily assimilated. The essential machinery must be constructed with care, for although the ideas may have solid geometric motivation, the level of abstraction and complexity can lead to confusion and a lost sense of direction. One must scrupulously avoid the tendency to view the constructions as the objective rather than as the tools to be applied. This transition is best made gradually, with ample explanations and examples. The patience and perseverance required are amply rewarded since a foundation is established for developing more sophisticated methods with applications far beyond these initial steps. Additionally, analogous constructions and techniques have evolved in other branches of mathematics and have become part of the established repertoire.

Perhaps these are in part the reasons why an introductory course in algebraic topology appears in the mathematical curriculum at many institutions. This has not always been the case. As recently as twenty-five years ago such courses were quite rare. The discipline itself was already well established, benefiting from the research of some of the finest mathematicians of the time. But most of these scholars were originally trained in related fields and developed their interest in algebraic topology as a means of solving problems. Concomitantly, prior to 1950 only a limited number of basic treatises were available. In addition to Poincaré's classic (1895), there were books by Lefschetz (1942), Seifert and Threllfall (1934), and Aleksandrov and Hopf (1935), and the Colloquium lectures of Veblen (1931) and Lefschetz (1930 and 1942).

The spread of interest in the subject in the early 1950's was sparked by significant advances in research and sustained by the publication of the Cartan Seminar (1948–) and books by Lefschetz (1949), Eilenberg and Steenrod (1952), Aleksandrov (1956), Cartan and Eilenberg (1956), A. H. Wallace (1957), and Hilton and Wylie (1960). The first truly comprehensive textbook, Spanier (1966), still perhaps the most influential in the field, was written in such depth and generality as to make it very cumbersome for an introductory course. Other books began to appear that were more narrowly focused: Bourgin (1963), Schubert (1964), Franz (1965), Hu (1966), Fréchet and Fan (1967), Massey (1967), Greenberg (1967), Cooke and Finney (1967), and Artin and Braun (1969). This expansion in the literature continued through the last decade with introductory books by the reviewer (1973), Rinow (1975), Agoston (1976), Massey (1978), and Croom (1978), more comprehensive treatments by Dold (1972) and Switzer (1975), and those based in homotopy theory by Gray (1975) and Whitehead (1979). A unique addition was the student's guide prescribed by Adams (1972).

The present book is very much in the spirit of those published since the appearance of Spanier. It presents a thorough and detailed treatment of singular homology and cohomology theory, including external, cup, and cap products and applications to CW complexes and topological manifolds. In reading through it one cannot escape the impression that this is the result of years of teaching the subject; topics are developed carefully, and all the basic questions are asked, many are answered, and others are left for the reader to explore. There is deliberate coordination between this book and the author's popular previous text dealing with covering spaces, the fundamental group, and the classification of surfaces.

The first three chapters explore the motivation for the development of homology theory, its basic properties, and the immediate applications, including the relationship between $\pi_1(X)$ and $H_1(X)$. Chapters four through six apply these ideas to the homology of CW complexes and product spaces and expand the techniques to include homology with arbitrary coefficient groups. The last four chapters introduce cohomology groups and the products arising from the pairing with homology and apply these constructions to study duality in topological manifolds, the cohomology rings of projective spaces, and the Hopf invariant.

Since these topics are fairly standard among existing texts of comparable scope, one might look for characteristics that set this book apart. One need not look far. The most obvious difference is the use of singular cubical chains rather than singular simplicial chains. This requires only minor modifications in the usual proofs of the theorems, but, as the author points out, offers several advantages: n-dimensional cubes are more easily described than

n-dimensional simplices; the product of a cube and an interval is again a cube, facilitating the treatment of homotopies; both products and subdivisions are more naturally described for cubes than simplices. These advantages are balanced by the added nuisance that degenerate singular cubes must be factored out along with the bounding chains.

Throughout the book there is a heavy emphasis on motivation and historical background. This is particularly apparent in the first chapter, set aside exclusively for this purpose, which includes a careful explanation of the problems from analysis which led in the nineteenth century to the nascence of the fundamental concepts in homology. Once the basic ideas are introduced, they are methodically developed through stages of increasing complexity. For example, the homology of finite graphs and compact surfaces leads naturally to the homology of CW complexes. Even a formula which is easily stated, like the expression for the boundary of the product of two chains, is preceded by an analysis of the incidence numbers that arise between cells in the product of two regular CW complexes.

Perhaps more so than any other comparable text, this gives complete references to previous results and to the origins of methods that have become part of the folklore. For example, the reader is given directions for a detailed introduction to cohomology with compact supports (the Cartan Seminar) and for a description of the Cayley projective plane (mimeographed notes of Freudenthal) and the resulting Hopf map $S^{15} \to S^8$ (Steenrod's book). The text is supplemented by a larger than average number of exercises which are, in general, less difficult than those in Spanier. They vary from direct applications or extensions of the topics presented to developments that probe new areas. For example, an exercise in Chapter six introduces a method which leads to one construction of the Steenrod squaring operations in cohomology. The formal exercises are expanded considerably by the large number of results in the text left to the reader for verification.

A somewhat surprising bonus is the inclusion of an appendix presenting differentiable singular chains, the cochain complex of differential forms on a manifold, and a proof of deRham's Theorem giving an isomorphism between the resulting cohomology groups. The basic method of proof (due to Milnor) is analogous to the proof given for the Poincaré Duality Theorem, first establishing the result for open convex subsets of Euclidean space and extending the result in stages to finally apply to manifolds in general.

The only criticism that seems appropriate relates to the omission of some additional topics or to the brevity with which others are presented. Specifically, the nonsingular bilinear form arising in the cohomology ring of a closed oriented manifold is discussed, but no mention is made of the signature, or index, which is a very useful invariant in the applications of algebraic topology. Later the author manages to state an amazing number of results related to the Hopf invariant (some with proofs) in less than three pages of the text. It would seem that such a fascinating topic would warrant some additional intuitive explanation accompanied by abbreviated reasons for the many interesting facts.

For many years the author's previous book has been a standard for use in introductory courses or individual study. Its continuing availability was assured when it was reprinted in the Springer Series of Graduate Texts in Mathematics, No. 56. From every indication, this book should be received with comparable continuing enthusiasm. It is a welcome addition to the literature.

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Measurement theory: with applications to decisionmaking, utility, and the social sciences, by Fred S. Roberts, Encyclopedia of Mathematics and its Applications, vol. 7, Addison-Wesley, Reading, Mass., 1979, xxii + 420 pp., $24.50.

Although measurement theory is not widely known academically or within the mathematics community and claims no professional society or exclusive journal, it has developed into a cohesive body of significant proportions during the past few decades. As often happens in areas that undergo a period of intense development, measurement theory has been afforded treatment in several books, the most recent of which is the one under review. Roberts' volume was preceded by Pfanzagl's *Theory of measurement* (1968), Volume I of the *Foundations of measurement* (1971) by Krantz, Luce, Suppes and Tversky (with Volume II nearing completion), and Fishburn's more specialized *Utility theory for decision making* (1970). These earlier works are primarily technical renderings that emphasize the axiomatic approach to measurement. While Roberts also stresses axiomatics, *Measurement theory* devotes considerable space to applications.

Early work in the theory of measurement focused on empirical laws or phenomena in physics – and to a lesser extent perhaps in psychophysics, economics, and other disciplines – that could be represented numerically, and on the special properties of such representations. While empirical phenomena continue to inform and motivate the subject, recent contributions have centered on axioms for qualitative relational systems that enable mappings into numerical systems that preserve the relational structures of the qualitative systems. Major contributors include mathematicians and mathematically oriented investigators in psychology, economics, philosophy, and statistics. A significant proportion of the articles on measurement theory that have appeared in the past twenty years are in the *Journal of Mathematical Psychology*, *Econometrica*, *Psychological Review*, and the *Annals of Statistics*. Mathematically, measurement theory draws heavily upon algebra and functional analysis, and is involved in various ways with discrete mathematics, probability theory, and topology. The relatons used in its axioms are usually binary orderings, either complete or partial.