uniqueness result is very tidy: $P$ is unique, and $u$ is unique up to a positive affine (linear) transformation: $v$ serves in place of $u$ iff there are reals $\alpha > 0$ and $\beta$ such that $v = \alpha u + \beta$.

The preceding theories, plus others that involve difference measurement, product structures with additive and nonadditive representations, expected utility, subjective probability, and measurement based on partial orders and binary choice probabilities, are discussed by Roberts. Because *Measurement theory* is designed to introduce the reader to the subject without getting bogged down in mathematical details, longer proofs that are available elsewhere are not repeated. The presentation is carefully developed and is mathematically rigorous in the best sense of that phrase. At the same time, the book proceeds at a relaxed and readable pace that reflects substantial concern and expertise on the author's part to communicate with readers not previously conversant in measurement theory.

As an introduction to the axiomatic approach to measurement theory, the book succeeds well. Its value as a general introduction to measurement is considerably enhanced by numerous examples from the behavioral and social sciences. One chapter is devoted to psychophysical scaling, and there are discussions of application in energy, air-pollution, and public health.

Roberts includes a wealth of exercises that extend the theory and suggest a variety of potential applications. He has used parts of the book in an undergraduate course in mathematical models in the social sciences, and most of the book with first-year graduate students in mathematics. While I believe that *Measurement theory* is well suited for introductory courses as well as informal learning situations, it should also prove useful as a reference source for people doing research in measurement theory.

All told, I feel that Roberts' book is superbly well done, and that it should serve handsomely as *the* introduction to the theory of measurement for many years to come.

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*Étale cohomology*, by J. S. Milne, Princeton Univ. Press, Princeton, N.J., 1980, xiii + 323 pp., $26.50.

A journalist once asked Sir Arthur Eddington (or perhaps it was Rutherford, the story is doubtless apocryphal anyway) whether he was one of only three men in the world who understood Einstein's theory of relativity. "And who," came the reply, "is the third?"

Here is a similar story I can vouch for personally. About a week after P. Deligne proved the last of the Weil conjectures several years ago (more about these in a moment) I received through the good offices of a friend who was in France at the time some fifty pages of detailed notes on the proof. This obviously was a hot item. I was visiting a major North American university, so I offered the chairman, himself a number theorist, the notes for Xeroxing.

"Oh no," he said, "no one in this department would understand them anyway."

How sad that modern science has come to this pass. Not only are we misunderstood by the world at large, but so few of us actually understand the greatest achievements in our own fields. This brings me to étale cohomology, that extraordinary unification of arithmetic and topology which must rank as one of the triumphs of twentieth century mathematics. It was conceived by Grothendieck and realized by Artin, Deligne, Grothendieck, and Verdier in 1963. (The image of mad scientists rubbing their hands with glee while electricity flashes against the darkened sky and the monster slowly raises himself to the sitting position on the table . . . is perhaps exaggerated.) Since then, in arithmetic it has led to the proof of the Weil conjecture (Deligne), in algebraic topology it was central to the proof of the Adams conjecture by Quillen and Sullivan, in group theory it enabled Deligne and Lustig to give a unified construction of all characters of finite Chevalley groups, . . . . In short, this is a gal all the boys should be pursuing. If they're not (and they're not) perhaps it's because she's misunderstood, or more precisely because she's not understood at all. Enter the good doctor Milne with an impressive tome revealing all, including diagrams of her private parts. But before we take a peak at what Professor Milne has to show us, let's recall briefly the motivation as formulated by Weil in about 1950.

Suppose we have a variety $X$ defined over the finite field (Galois field) of $q$ elements $\mathbf{F}_q$. For example, $X$ might be the zeros of a polynomial $f$ with coefficients in $\mathbf{F}_q$,

$$X: f(T_1, T_2,, \ldots, T_n) = 0.$$

One would like to know how many points $X$ has with values in $\mathbf{F}_q$. (In the example, this amounts to asking how many solutions $f$ has with the $T_i$ in $\mathbf{F}_q$.) Weil's insight is that we should view these points as fixed points of a certain endomorphism, the frobenius, acting on the variety $\overline{X}$ obtained by extending the scalar field from $\mathbf{F}_q$ to the algebraic closure $\overline{\mathbf{F}}_q$. Again in the example, the set of solutions $(t_1, \ldots, t_n)$ to $f = 0$ with $t_i \in \overline{\mathbf{F}}_q$ is stable under the automorphism $F$: $(t_1, \ldots, t_n) \mapsto (t_1^q, \ldots, t_n^q)$, and the set of fixed points of this map is precisely the set of solutions in $\mathbf{F}_q$.

Now Lefschetz, in a topological context, had established that under suitable transversality hypotheses, the number of fixed points of an endomorphism $F$ of a topological space $X$ was given by the alternating sum of the traces of $F^*$ acting on the Betti groups $H^*(X)$.

Weil remarked that what was needed was some analogue of the Betti groups in the context of our variety $\overline{X}$. Moreover, if one wanted estimates (number theorists love estimates) on the number of $\mathbf{F}_q$-points of $X$ (resp. $\mathbf{F}_q$-solutions of $f$) one best have some idea of the absolute values of the eigenvalues of $F$ as it acted on these pseudo Betti groups. Weil formulated some precise conjectures and proved them for the case of curves ($f = f(T_1, T_2)$, a polynomial in two variables). Some years later, Grothendieck et al. constructed a suitable Betti theory, étale cohomology, and subsequently

Deligne established the all important Weil conjecture that the eigenvalues of $F$ acting on $H^i(\overline{X})$ had absolute values $q^{i/2}$.

For example, when $X$ is a complete nonsingular curve of genuS $g$, one has $H^0(\overline{X})$, $H^1(\overline{X})$, and $H^2(\overline{X})$. $H^0$ and $H^2$ are one dimensional and $F$ acts by multiplication by 1 and $q$ respectively. $H^1(\overline{X})$ has dimension $2g$ and the Weil conjecture says that the eigenvalues of $F$ have absolute value $q^{1/2}$. Thus if $N$ is the number of $\mathbf{F}_q$-points of $X$, one finds

$$|N - 1 - q| \leqslant 2gq^{1/2}.$$

But what is this thing called étale cohomology? The problem with doing topology in arithmetic context is that there is no good algebraic notion of an open neighborhood of a point. On the other hand, as Zariski and Abhyankar established, there is a reasonable notion of an algebraic fundamental group, lying somewhere between a topological fundamental group and an arithmetic galois group. Grothendieck's insight was that one could build the whole machinery of algebraic topology on the notion of covering. Open sets no longer sit *in* the space, rather they lie *over* the space. Combine this insight with a peck of homological algebra and out comes étale cohomology.

Actually, there is one other point which should be mentioned. Pathologies can occur if one works with infinite coverings. It is best, as in galois theory, to work with finite extensions. Thus, in the first instance, one defines cohomology groups with finite coefficients,

$$H^i_{\text{étale}}(\overline{X}, \mathbf{Z}/n\mathbf{Z}).$$

(It is also well to stick to the case when the coefficient group has order prime to the characteristic $p$ of the field of definition of $X$. The $p$-adic theory has curious properties and is best left to the experts.) One then defines an $l$-adic theory

$$H^i_{\text{étale}}(\overline{X}, \mathbf{Z}_l) = \varprojlim_r H^i_{\text{étale}}(\overline{X}, \mathbf{Z}/l^r\mathbf{Z}).$$

Finally, after a couple of drinks things tend to get sloppy and one begins to neglect torsion, defining

$$H^i_{\text{ét}}(\overline{X}, \mathbf{Q}_l) = H^i_{\text{ét}}(\overline{X}, \mathbf{Z}_l) \otimes_{\mathbf{Z}} \mathbf{Q}.$$

Now to work! Or, in other words, to quote from the book under review (Chapter I, paragraph 1, from the top).

"Recall that a morphism of schemes $f\colon X \to Y$ is affine if . . . ." This is (clearly) a serious scholarly work, very much in the Grothendieck school with emphasis on sheaves, exact sequences, homological algebra, and what might be called the geometry of arrows (Corollary (1.10) on p. 6 for example: "Any proper, quasi-finite morphism $f\colon Y \to X$ is finite."). An amusing point: in leafing through the collected works of Weil (who in some sense started it all) I am unable to find a single exact sequence or commutative diagram. The reader is invited to compare this with the work under review (or indeed with any of the published work of either author or reviewer). It would be interesting to see more clearly the shift in mathematical philosophy which must underlie this shift in notation.

The first three chapters discuss étale morphisms, sheaf theory, and cohomology. The first two topics are covered rigorously, if not in a totally self-contained fashion. Milne recommends, for example, the book of Atiyah-Macdonald on commutative algebra and refers to various technical results proved there. The section on sheaves treats with some care the various functors on sheaves $f^*, f_*, f^!, f_!$ associated to a closed immersion $f\colon Y \to X$. The important notion of constructible sheaf is postponed and appears for the first time somewhat incongrously in the chapter on curves and surfaces.

The third chapter, on cohomology, discusses derived functor and Čech cohomology as well as change of topology and principal homogeneous spaces (including some discussion of principal homogeneous spaces for the flat topology). I must criticize somewhat the treatment of cohomology with compact support. Suppose $X$ is a variety which is included as an open subscheme in a complete variety $\overline{X}$,

$$X \overset{i}{\hookrightarrow} \overline{X},$$

and $F$ is a sheaf on $X$. The extension by zero $i_!F$ is the sheaf on $\overline{X}$ whose stalks at points of $X$ coincide with those of $F$ and whose stalks are 0 on $\overline{X} - X$. One defines the cohomology with compact support $H_c^*(X, F)$ to be the cohomology of $i_!F$. As in the classical case of singular cohomology there are two important facts:

(i) When $F$ is the restriction of a sheaf $\overline{F}$ on $\overline{X}$, there is a long exact sequence

$$\cdots \to H_c^i(X, F) \to H^i(\overline{X}, \overline{F}) \to H^i(\overline{X} - X, \overline{F}|\overline{X} - X) \to H_c^{i+1}(X, F) \to \cdots.$$

(ii) When $F$ is a torsion sheaf, the groups $H_c^*(X, F)$ are functorial in $X$ and independent of the choice of compactification $\overline{X}$.

Property (i) is easy, but (ii) is proved using base change, which is a deeper result for étale cohomology. Milne defines these cohomology groups with compact support in Chapter III, mentioning that he will establish independence of $\overline{X}$ in Chapter VI. Meanwhile the groups are used extensively in Chapter V (admittedly only for smooth curves, where it is possible to define a canonical compactification). Since results from Chapter V would go into a complete proof of base change, there is a certain feeling of circularity.

I am also unhappy with chapter IV on the Brauer group. The Brauer group plays an important role in the development of étale cohomology through the medium of Tsen's theorem, which implies that the Brauer group of the function field of a curve vanishes. From this, one deduces vanishing for the cohomology of a curve in degrees greater than 2. Milne doesn't prove Tsen's theorem (although the proof is both short and elegant), but instead devotes a whole chapter to aspects of the Brauer group much less central to the theory.

Chapter V discusses cohomology of curves and of surfaces. I particularly liked the discussion for surfaces, including vanishing cycle theory and a proof by Artin of Castelnuovo's criterion for rationality of a surface.

Chapter VI is the heart of the book, containing the base change theorem, Poincaré duality, and rationality for zeta and $L$-functions, as well as other topics. These are all carefully treated with the exception of base change. Perhaps I should explain, since the phrase has come up several times, that base change refers to the analogue in étale theory of the result in singular theory that for a proper map of reasonable topological spaces $f\colon Y \to X$, one has for $x \in X$

$$H^*(f^{-1}(x)) \cong \varinjlim H^*(f^{-1}(U))$$

where $U$ runs through neighborhoods of $x$. (The proof follows from the existence of neighborhood retracts of $f^{-1}(x)$ in $Y$.) Given the importance of this result, Milne's treatment is much too rapid. I recommend instead the proof in Deligne's Springer Lecture Notes SGA $4\frac{1}{2}$.

In sum, the author has done a tremendous service by organizing the material in a careful and united way which makes it possible for serious students to learn. In its way, the terse and brilliant account of the theory by Deligne (who disposes of the whole business in 65 pages) in SGA $4\frac{1}{2}$ is unexcelled. On the other hand, having watched graduate students trying to make sense of the many details, I can testify to the need for a book like this. Having tried to teach a course in the subject, I can testify to the achievement it is to write one.

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*Curves on rational and unirational surfaces*, by M. Miyanishi, Tata Institute of Fundamental Research, Bombay, 1978, Narosa Publishing House, New Delhi, 1978, 307 pp., $9.90.

Traditionally, algebraic geometry has meant the study of projective varieties, with its richest results having been produced for curves and surfaces in projective spaces. So, it was not always clear how deeply algebraic geometry was related to commutative algebra. Until rather recently, one might almost perfunctorily start out with affine varieties as zeros of ideals in a polynomial ring, but as soon as one got serious one would switch over to projective varieties and geometric arguments. Indeed, the great Italians (Castelnuovo, Enriques, Severi, . . . ) appeared oblivious to commutative algebra while developing their immensely successful algebraic surface theory. Even Hilbert,