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Vector bundles on complex projective spaces, Progress in Mathematics, vol. 3, by Christian Okonek, Michael Schneider and Heinz Spindler, Birkhäuser, Boston, 1980, vii + 389 pp., $18.00 paperback.

This book is an introduction to the basic theory and classification of holomorphic vector bundles on complex projective spaces. A holomorphic vector bundle on a complex manifold M is just what you would expect: It is a locally trivial fibre space E over M with fibre $\mathbf{C}^r$ and with transition functions which are holomorphic on the base. The dimension r of the fibre is called the *rank* of the bundle.

Smooth vector bundles (that is, with C^∞ transition functions) have been known for a long time in differential geometry. They can in principle be classified by certain characteristic classes (in the case of $\mathbf{C}^r$-bundles their Chern classes) and some homotopy invariants. The study of holomorphic vector bundles is more recent, and the classification problem is of an essentially different nature because of the extra structure imposed by holomorphic functions. Once the topological type of the underlying smooth vector bundle has been fixed, one finds in general continuous families of nonisomorphic holomorphic bundles. The parameter spaces of these families are called *moduli spaces*. The classification problem thus consists of determining which smooth $\mathbf{C}^r$-bundles carry a holomorphic structure, and then describing the moduli space of the possible holomorphic bundles.

Holomorphic vector bundles on compact Riemann surfaces were studied extensively beginning in the 1960's. Then attention turned to higher-dimensional manifolds and in particular, there has been a big spurt of recent activity concerning holomorphic vector bundles on complex projective spaces, the subject of this book. The extent of this activity can be judged from the bibliography of this volume, which contains 138 items, about half of which date since 1977.

Why is this subject suddenly so popular? I see three principal reasons. One is the ties with classical algebraic geometry of varieties in projective space. Note first, by a well-known theorem of Serre ("géométrie algébrique et géométrie analytique") that every holomorphic vector bundle on complex projective space carries a unique algebraic structure. Thus the classification problem is equivalent to the classification of algebraic vector bundles, so can be regarded as a problem in algebraic geometry. The connection with classical algebraic geometry can be illustrated best in the special case of rank 2 bundles on $\mathbf{P}^3$. In general, a global section of such a bundle will vanish along a subset of $\mathbf{P}^3$ of codimension 2, namely a curve. Thus the study of such bundles is intimately related to the late 19th century problem of classifying all algebraic curves in $\mathbf{P}^3$–a problem, by the way, which in many respects is still open.

A second reason for current interest in vector bundles on complex projective spaces is the connection with physics. The so-called gauge theories are essentially the study of certain (smooth) vector bundles on complex manifolds associated with the real world, and thanks to the Penrose twistor program, some of the partial differential equations which arise from mathematical physics can be translated into questions about *holomorphic* vector bundles on $\mathbf{P}^3_{\mathbf{C}}$. This connection is explained for example in Atiyah's talk at the International Congress of Mathematicians in Helsinki. While I cannot speak about the significance of this connection for physicists, there is no doubt that it has raised many interesting questions for mathematicians, and as such has provided a great stimulus to the theory.

A third reason for current interest in this subject (a reason which is indeed necessary for the vitality of any discipline) is the presence of intriguing unsolved problems. Just to mention two, it is not known whether there are any indecomposable rank 2 bundles on $\mathbf{P}^n$ for $n \geqslant 5$, and it is not known whether the moduli space of rank 2 mathematical instanton bundles on $\mathbf{P}^3$ with fixed Chern classes is connected.

One of the authors of this volume (M. Schneider) gave a report in the Séminaire Bourbaki (1978/79, exposé 520) on the current state of knowledge concerning vector bundles on projective spaces. Indeed, that report could profitably be read as an introduction to this book. The book arose out of a series of subsequent lectures by Schneider. It gives necessary background and many of the basic results of the subject, assuming only a standard knowledge of complex manifolds or algebraic geometry with sheaf theory and cohomology. In particular, it contains extensive discussion of the *splitting type* of a bundle restricted to a line (a $\mathbf{P}^1$) in $\mathbf{P}^n$, *uniform* bundles (those for which the splitting type is the same on all lines), *stable* bundles and their variety of moduli, and the *monads* introduced by Horrocks. One particularly nice feature of the book is that every section ends with historical comments, further results, and open questions. This brings the reader up to date and provides a guide for further work. There are practically no new results in this book. On the other hand, the authors have gathered their material from many sources, which is a great service to the reader. Unfortunately, they sometimes omit part of a proof (for example in Vogelaar's theorem, p. 136) so that the reader will have to consult the original paper anyway. My only other complaint is that the authors always use the language of complex manifolds and holomorphic sheaves, and do not point out that everything they say makes sense in abstract algebraic geometry over an algebraically closed groundfield k. In fact, with the exception of Chapter II §2 on the generic splitting type of a semistable bundle, there is no need even to suppose the groundfield of characteristic 0.

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Elements of soliton theory, by G. L. Lamb, Jr., Wiley, New York, 1980, xii + 289 pp., $29.95.

Until comparatively recently in the history of science, mathematics and physics lived in close relation. The advance of physics suggested the development of new mathematics, and conversely the progress of mathematics fed into the way physicists thought about nature. This contact remained strong until the 1920's, when it began to ebb, reaching a low in the 1950's. What is beginning to bring us together again are two striking developments of science in the 1960's–the successful application of Lie group theory and differential geometry to elementary particle physics and the theory of *solitons*.

The first topic was the result of efforts by a thundering herd. In contrast to this 'big science', the theory of solitons was the remarkable creation of a small group–Kruskal, Zabusky, Gardner, Greene and Miura–working at Princeton (but not in the mathematics or physics department!). That the roots of the theory lie in the 19th century work on hydrodynamics of Scott Russell and computer studies of solutions of nonlinear differential equations by Fermi, Pasta and Ulam in the 1950's has been recounted many times, [**1**, **2**], and will not be repeated here.

The Princeton group produced a complex of original ideas that has had a remarkable success and influence in contemporary science. Their theory was developed with the traditional tools of the physically-minded applied mathematician: classical differential equation theory, the mathematics of quantum mechanics, scattering theory, etc. A remarkable quality of their work is that it linked 19th century analysis and geometry with some of the most modern parts of functional analysis, differential, and algebraic geometry.

This book is an admirable attempt by a physicist-applied mathematician to present an introductory account of the theory of solitons from some intermediate point on the scale of mathematical sophistication. Lamb explicitly excludes an attempt to describe the geometric or Lie theoretic side of the theory, although his own research work is in this direction. Since a mathematician who comes to the subject cold and tries to read this book or the original literature [**3**, **4**] will probably have difficulty seeing the forest for the trees, and the 'geometric' point of view is valuable precisely for the overall perspective it