always assumed). The heart of the book is two long chapters on classification theory: 160 pages on endomorphisms and classification by power series methods, and 110 pages on Dieudonné modules (done *via* curves, and including the "tapis de Cartier"). The applications then take just 50 pages, followed by 40 pages sketching the dual approach through hyperalgebras.

The one objection to the book, perhaps, is that it is sometimes a bit too reminiscent of the eighteenth century: my own tolerance for power series computations gives out before the author's does. His bent is also away from my own main interest, the connections with finite group schemes and p-divisible group schemes and abelian varieties; for this any interested reader should consult [**6**]. The three applications, though they should be clear to specialists, presume enough background that the general reader may have trouble appreciating them. Still, the book will clearly be a standard and valuable reference work. It has good bibliographical notes and a bibliography essentially complete through 1976+ . Hazewinkel has also nicely made the first section of each chapter a self-contained introduction and summary, so that anyone reading just these parts can get a fair first impression of the material. This might well be worth doing, if only to see what a sophisticated theory can arise from the simple question of which power series satisfy $f(x, f(y, z)) = f(f(x, y), z)$.

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Ramsey theory, by Ronald L. Graham, Bruce L. Rothschild, and Joel H. Spencer, John Wiley and Sons, Inc., New York, 1980, ix + 174 pp., $21.95.

In recent years there has been among mathematicians an explosion of interest in discrete mathematics. In particular, an enormous number of Ramsey type results have been published during the last ten years. Thus, it is appropriate that a book should be published on Ramsey theory.

Although the scope of the book is broad, covering all of the main areas of the theory, the authors do not attempt to make the book an encyclopedia of known theorems, proofs and conjectures. Instead, they have chosen to give the reader a feel for the present state of the subject and its possible future directions by concentrating on central results when possible and representative results otherwise, and on important open questions. Special attention is given to demonstrating techniques that are useful in proofs and constructions, discussing the relationships among, and consequences of, major theorems, and illustrating these with appropriate examples. Typically, after a precise statement of a central theorem, there is a simple and illuminating example of a special or small order case which leads to at least one formal proof of the theorem. This is in turn followed by open questions. All of the results mentioned are carefully and conveniently referenced, although the bibliography is not extensive because of the nature of the book. This is definitely a very current research book, but the style of exposition has the flavor of a text book. Difficult proofs are not made easy but they are made digestible. The examples are invaluable to the reader and, I think, one of the outstanding features of the book.

The content of the book is centered around six theorems which the authors call the "Super Six". Because of their importance and the simplicity of their statements, they are listed. In the statements that follow, let k, l and r be fixed positive integers, $n_i = n_i(k, l, r)$ sufficiently large positive integers, $[n]$ the first n positive integers, and $[n]^k$ the set of k-subsets of $[n]$. The "Super Six" are:

(1) RAMSEY'S THEOREM. *If for* $n \geqslant n_1$, $[n]^k$ *is r-colored, then there is a monochromatic* $[l]^k$.

(2) VAN DER WAERDEN'S THEOREM. *If for* $n \geqslant n_2$, $[n]$ *is r-colored,* $[n]$ *contains a monochromatic arithmetic progression of length l.*

(3) SCHUR'S THEOREM. *If for* $n \geqslant n_3$, $[n]$ *is r-colored, then* $x + y = z$ *has a monochromatic solution in* $[n]$.

(4) RADO'S THEOREM. *If for* $n \geqslant n_4$, $[n]$ *is r-colored, the system* $c_1x_1 + c_2x_2 + \cdots + c_lx_l = 0$ *has a monochromatic solution iff some nonempty subset of the* c_i *sum to* 0.

(5) HALES-JEWETT THEOREM. *If for* $n \geqslant n_5$, *the n-dimensional cube* $\{(x_1, \ldots, x_n)\colon x_i \in [k], i \in [n]\}$ *is r-colored, there is a monochromatic "line".*

(6) GRAHAM-LEEB-ROTHSCHILD THEOREM. *If for* $n \geqslant n_6$, *the k-dimensional subspaces of an n-dimensional vector space over a fixed finite field are r-colored, then there is an l-dimensional subspace with all of its k-dimensional subspaces having the same color.*

In the spirit of the book a simple small order example would be appropriate. Possibly the most widely known consequence of Ramsey's theorem is the special case when $l = 3$ and $k = r = 2$. It is often expressed as follows: In any collection of 6 people, there are either 3 people who mutually know each other or there are 3 people who are mutually strangers. Thus, $n_1(2, 3, 2) = 6$. A formulation closer to the terminology used in the statement of the theorem would be the following. Consider a complete graph K_6 (the graph consists of 6 points and 15 lines joining the 15 pairs of points). If the lines of K_6 are 2-colored (i.e., partitioned into 2 sets) in any way, then there is a monochromatic K_3 (triangle).

Proofs of Ramsey's theorem are given, followed by a worthwhile brief glimpse into the background of F. P. Ramsey, and a reproduction of his original proof of an infinite version of (1). The importance of the rediscovery and application of this theorem by P. Erdös and G. Szekeres is noted along with accompanying interesting historical facts. Proofs, implications, special cases and relationships among the six theorems, interspersed with historical notes and occasional levity, occupy the first half of the book. Extensions of these results are also discussed. An example of this is Szemerédi's extension of van der Waerden's theorem, which states that a sufficiently dense subset of positive integers contains arbitrarily long arithmetic progressions. No proof is given, but two proofs using very different methods are given of Roth's theorem, which is the special case of arithmetic progressions of length 3. No book on Ramsey theory would be complete without conjectures, especially ones by Paul Erdös. For the greedy, there is the following conjecture for which he offers $3000.

CONJECTURE. *If A is a set of positive integers such that $\sum_{a \in A} 1/a = \infty$, then A contains arbitrarily long arithmetic progressions.*

I thoroughly enjoyed the first 3 chapters of the book which covers the "Super Six" and their descendents. Each of the six theorems states the existence of a sufficiently large n such that a structure of size n contains an appropriate substructure. Thus, associated with each theorem and set of parameters is a smallest number with the desired property. For example in the case of van der Waerden's theorem, $W(k, r)$ is the smallest W such that when $[W]$ is r-colored, there is a monochromatic arithmetic progression of length k. One chapter is devoted to exact results for small order cases and to asymptotic bounds of these functions. Ramsey numbers have received the most attention, but even here there are few results. The difficulty of results of this type has motivated generalizations which are considered later in the book.

Theorems concerning the various generalizations of Ramsey's theorem could fill more than one book. In one chapter, the authors could do no more than highlight some representative results of this type. Selected theorems from Graphical Ramsey theory (finding monochromatic graphs which are not necessarily complete in a complete graph whose edges have been colored) and Euclidean Ramsey theory (finding monochromatic geometric configurations in an n-dimensional Euclidean space whose points have been colored) are given. Among other things in this chapter, the problem of finding induced monochromatic subgraphs in a graph whose edges have been colored is touched upon.

The nature of the content of the last chapter is considerably different. In fact, some parts are almost disjoint from the remainder of the book, but it is consistent with the basic philosophy of the book. Techniques from topological dynamics are used to prove a topological version of van der Waerden's theorem and of Hindmans theorem (which states that if N is finitely colored, then there is an infinite subset $S \subseteq N$ such that the set of all finite sums of distinct elements of S is monochromatic). Szemerédi's theorem (mentioned earlier) is shown to be a consequence of a theorem in ergodic theory by Furstenberg. One of the most interesting results in this chapter is the one by Harrington and Paris that the Ramsey type property (PH) on the integers is unprovable in Peano arithmetic. Property (PH) is the statement: For any positive integers n, k, and r, there is an m such that in any r-coloring of $[n, m]^k$ there is a monochromatic large set (a set $S \subseteq N$ is large if $|S| > \min(S)$). The authors complete the book with some selected results from the Ramsey theory of infinite sets.

One of the most miserable tasks of any author is to proofread the text. In this case, the task was taken seriously. The errors are minimal and most will be transparent to the reader.

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Spectral theory of ordinary differential operators, by Erich Müller-Pfeiffer, Ellis Horwood Series Mathematics and its Applications, Wiley, New York, 1981, 246 pp., $56.95.

With the study of Hilbert space having evolved into a challenging end in itself, it has become quite possible to overlook the connection between this area of mathematics and its underlying problems in differential equations. Such connections date back at least to the 1830's when Sturm and Liouville sought to generalize the idea of Fourier (sine) series to more general expansions in terms of the eigenfunctions associated with Sturm-Liouville equations of the form $-(pu')' + qu = \lambda\rho u$. While concepts such as "orthogonality" and "Fourier coefficient" generalize readily, questions regarding the convergence of these generalized Fourier series proved to be more difficult. Liouville's claims of rigor notwithstanding, it seems that questions of convergence were not properly formulated, much less rigorously resolved, until after 1900 [**2**].

The Hilbert space formulation of convergence provided more than rigor. It provided a context in which this question (and its trigonometric series specialization) has a simple and elegant solution, replete with necessary and sufficient conditions for such convergence. It also made the study of selfadjoint realizations of the formal operator $-\frac{d}{dx}p\frac{d}{dx} + q$ in the Hilbert space $L^2_\rho(I)$ a subject of profound importance.

E. Müller-Pfeiffer's *Spectral theory of ordinary differential operators* continues this study of selfajdoint realizations of Sturm-Liouville operators in terms of their natural generalization

$$a[\cdot] = \sum_{k=0}^{n} (-1)^k \frac{d^k}{dx^k} a_k(x) \frac{d^k}{dx^k}; \qquad a_n(x) > 0.$$

The coefficients are assumed to belong to an appropriate Sobolev class on an interval I of the form $x_0 < x < \infty$, where $x_0 \geqslant -\infty$. It is in this singular case,