Spectral theory of ordinary differential operators presents a very current view of an important area of mathematics from the vantage point of one of the experts in the field. It is not encyclopedic, in the sense of Dunford and Schwartz [3], but instead presents a rather personal view. When it comes to a choice between developing the theoretical background vs. displaying the details of what is required to apply the theory, Professor Pfeiffer will tend to provide references to the theory but present the detailed calculations—ones which frequently represent his own signficiant contributions to the field.

The background and motivation for such hard work may have to be found in the texts and references to which the author frequently refers us. But since it very knowledgably brings the reader into contact with the techniques which are required to increase our understanding of such spectral theory, Professor Müller-Pfeiffer's book represents a valuable contribution to further research in this area.

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KURT KREITH

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Combinatorial algebraic and topological representations of groups, semigroups and categories, by Ales Pultr and Věra Trnková, North-Holland Mathematical Library, Amsterdam, 1980, vi + 372 pp., $48.75.

The present subject is, of course, not much like group representation theory. But that can be said of most new subjects. Here there are two fundamental differences. First, one does not find distinguished representations, nor does one classify nice representations: the problem is existence. Think of Beltrami's Euclidean model of hyperbolic geometry. It proves (a) relative consistency, and (b) adequacy of the Euclidean theory of rather simple curves for formulating the Lobačevskian theory of straight lines. In the opposite direction, second, these are sharper representations: a group is represented as the group of *all* automorphisms of some structure; a category is represented by a number of structures of the same type and all their structure-preserving maps.

One might classify types of structure by their capacity for representing each other. If the familiar terms "hard", "soft", were to be given precise meanings, we might say "classify types of structure by their softness". What has actually been developed in about fifteen years is a theory of universally soft structures which can represent anything.

In the triad "groups, semigroups, and categories", the categories dominate. Certainly the facts that a group G is isomorphic with (1) a group of permutations of G, and precisely (2) the group of all permutations commuting with right translations, are the beginning; and the labor of proving universality for a structure type may be almost all local-universality for semigroups. But the theory is about full embeddings of categories.

The theory was ready to begin after Eilenberg and Mac Lane extended (1): a small category is isomorphic with a category of sets and functions, and the reviewer extended (2): a small category $\mathcal{C}$ is isomorphic with a full subcategory of the algebras of a certain type varying with $\mathcal{C}$. (Indeed, that observation was applied also to a lot of large categories **[9]**.)

That is where the questions came from. The techniques came rather from Frucht, Sabidussi, de Groot **[4, 5, 11]** on representing an arbitrary group as the group of all auto (-homeo)-morphisms of a graph, or a topological space, or a ring. Graphs first. Graphs remain first; and, of course, this line like the other goes back to Arthur Cayley.

In a series of papers culminating in **[8]**, in 1963–66, Hedrlin and Pultr showed that directed graphs, or algebras with at least one binary operation, or algebras with at least two unary operations, contain fully copies of (a) any small category, and much more, (b) any finitary variety of algebras. A further surprise was Kučera's result (announced at the Nice Congress by Hedrlin **[6]**) that if Ulam numbers come to an end (for some m, m-additive two-valued measures are completely additive), then (b) implies containing fully an isomorphic copy of any category which can be faithfully represented by sets and functions. This property, *full universality*, is perhaps as much universality as one could ask for. Without the (very modest) assumption on Ulam numbers, several rather simply defined categories have it, but seemingly no category that is the favorite subject of a nonempty family of mathematicians. However, the conceivably weaker property (b)—*alg-universality*—is a proven property of semigroups **[7]** and of integral domains of characteristic 0 **[3]**.

One could ask for more universality. ("I can call demons from the vasty deep"—Glendower.) One might ask for a concrete category $\mathcal{C}$ of sets and functions, $\mathcal{C} \to \mathcal{S}$, such that all reasonable such categories $\mathcal{B} \to \mathcal{S}$ can be fully embedded in it by a functor $\mathcal{B} \to \mathcal{C}$ (over 1: $\mathcal{S} \to \mathcal{S}$). This is quite unreasonable. The authors' explanation, p. 14, would easily fit here, but it doesn't seem appropriate. ("Why, so can I, or so can any man. But will they come when you do call them?"—Hotspur.) Accordingly some weaker versions of concrete universality have been studied, the best being the one that simply admits any functor $\mathcal{S} \to \mathcal{S}$. One has still to describe the "reasonable" concrete categories $\mathcal{B} \to \mathcal{S}$. It turns out that this can be done so that there exist reasonable universal (*strongly universal*) concrete categories; that is a new theorem of Kučera, first published in this book. "Reasonable" is about the most one can say: the most familiar established example (and that one only modulo Ulam numbers) is partially ordered sets. Pultr and Trnková showed much earlier **[10]** that no variety of algebras can be strongly universal.

This book has, of course, no competitor. Fortunately it is highly readable. A 20-page introduction sketches the main ideas and indicates where the hard work will lie. Then 100 necessarily dry pages do it; here one is usually concerned with categories little known outside this field, especially with

$S(P^+)$. The next 160 pages treat well-known categories and how $S(P^+)$ can be embedded in them. The final chapter, 25 pages, presents strong universality theory. A most welcome 50-page appendix treats Cook continua, which are essential for showing that there is any universality in topology. (Cook's papers [**1**, **2**] are much shorter—too much.) Finally, there is a brief survey of what happens if there exists a proper class of Ulam numbers.

The book jacket claims that no previous knowledge of category theory is assumed, and not much of other things. These claims are much better justified than such claims usually are.

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JOHN ISBELL

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A history of the calculus of variations from the 17*th through the* 19*th century*, by Herman H. Goldstine, Studies in the History of Mathematics and Physical Sciences, vol. 5, Springer-Verlag, New York-Heidelberg-Berlin, 1980, xi + 410 pp., $48.00.

During my term at Cornell University my then colleague Mark Kac was fond of a quip which he had heard from Steinhaus who had heard it from Lichtenstein who was quoting Boltzmann: "elegance should be left to shoemakers and tailors". The classical single integral problems of the calculus of variations, the subject of this book, made a difficult and dirty field that even Bourbaki cannot make elegant. Possibly this is why the recent two volume