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BOOK REVIEWS

Emmy Noether, 1882–1935, by Auguste Dick, translated by H. I. Blocher, Birkhäuser Boston Inc., Cambridge, Mass., 1981, xiv + 193 pp., $12.95.

This book is an excellent translation from the German book with the same title which appeared as a supplement ('Beiheft Nr. 13') of the Journal 'Elemente der Mathematik' in 1970. The author's 'Nachwort' followed by 'Zeittafel' was moved to the very beginning ('Acknowledgements' and 'Chronology'). The publication list and the list: 'doctoral dissertations completed under Emmy Noether' was moved to Appendix A, to be followed by the list of obituaries (Appendix B), and, in addition, by a list: 'Academic Ranks and Terms (with English equivalent or explanation)' as Appendix C and at the end a five page index of names. In between we find again the Introduction followed by a biography of Emmy Noether subdivided into: The Erlangen Period (1882–1915), The Göttingen Period (1915–1933), Bryn Mawr and Princeton (Fall 1933–Spring 1935), followed by the obituary of Emmy Noether by B. L. van der Waerden [Mathematische Annalen **111**(1935), 469–474] in English translation and the Memorial Address 'Emmy Noether' delivered by Herman Weyl in Goodhart Hall, Bryn Mawr College, on April 26, 1935, and published in Scripta Mathematica **3**(1935), 201–220. To this is added an English rendition of the Address delivered by the President of the Moscow Mathematical Society, P. S. Alexandrov, on September 5, 1935 (published in Proceedings of the Moscow Mathematical Society 1936, 2): 'In Memory of Emmy Noether'.

A large number of photos has been added to the English version.

Whenever I discuss Emmy Noether's life and work with my American colleagues invariably a number of questions turn up which deal with her as if she were a living person. Auguste Dick's book contains many valuable hints where to look for the answer, as will be evident from the following quotations.

Is it true that E. Noether 'converted' from purely formal algebra to 'modern' conceptual algebra?

> (p. 16) 'Under Gordan's influence Emmy Noether wrote a paper based on the theory of invariants, entitled, On the construction of the system of forms for the ternary biquadratic forms ("Über die Bildung des Formensystems der ternären biquadratischen Form").'

> (p. 17) 'The subject of this dissertation as well as its treatment correspond entirely to Gordan's interest. They do not indicate in any way the course that the author's thinking later took toward purely abstract algebra. Emmy Noether herself later referred to her thesis, as well as to several

> consecutive papers on the theory of invariants, as "crap". In 1932 she declared that as far as she was concerned, her dissertation was forgotten; in fact she couldn't even remember in which volume of Crelle's journal it had been published.'

Was there any person or event helping E. Noether to find her true vocation in mathematics?

> (pp. 23–24) 'In 1910 Gordan retired from his position as *Ordinarius*. His successor, Erhard Schmidt (1876–1959), had little impact on Erlangen. Schmidt was followed by Ernst Fischer (1875–1954); it is he who became a true mentor to Emmy Noether. With him she could "talk mathematics" to her heart's desire. Although both lived in Erlangen and saw each other frequently at the University, a large number of postcards exist from E. Noether to E. Fischer, containing mathematical arguments. Looking over this correspondence, one gets the impression that immediately after a conversation with Fischer, Emmy Noether sat down and continued the ideas discussed in writing, whether so as not to forget them, or whether to stimulate another discussion. Ernst Fischer has succeeded in carefully preserving these communications through all the havoc of war. The correspondence extends from 1911 to 1929 and is most frequent in 1915, just before Emmy Noether moved to Göttingen and Ernst Fischer was drafted by the military. There can be no doubt that it was under Fischer's influence that Emmy Noether made the definite change from the purely computational distinctly algorithmical approach represented by Gordan to the mode of thinking characteristic of Hilbert.'

Was there resistance to E. Noether's rise in the ranks right from the beginning of her Göttingen career?

> (p. 31) 'On November 9, 1915, in an effort to obtain *Habilitation*, Emmy Noether gave a lecture before the Mathematical Society in Göttingen entitled *Über ganze transzendente Zahlen* (On transcendental integers). On this occasion she comments to Fischer, "Even our geographer came to hear it and found it rather too abstract; the faculty wants to make sure it's not going to be duped at the meeting by the mathematicians". Duped or not, the deal was never made; *Habilitation* was declared impossible because of "unmet legal requirements"! According to the Privatdozentenverordnung (regulations concerning Privatdozenten) of 1908, Habilitation could only be granted to male candidates. An appeal to

the Minister of Culture was rejected. Hilbert's objection (not positively authenticated but willingly repeated)—that he couldn't see why the gender of the candidate should be an argument against admission as a Privatdozent; after all, this was a university, not a bathing establishment—effected nothing.'

What happened later on?

(p. 168) 'Her appointment as Privatdozent in 1919 was only possible because of the persistence of Hilbert and Klein, who overcame some extreme opposition from reactionary university circles.'

(p. 50) 'The course of E. Noether's official career after her *Habilitation* in 1919 is as follows. On April 6, 1922, the Prussian Minister of Science, Art and Public Education presented Fräulein Dr. Emmy Noether with a document attributing to her the official title of *ausserordentlicher Professor*. The text of this document literally states that this act was being carried out " . . . with the understanding that this title does not signify any change in your present legal position. In particular, your relations to your faculty as they ensued from your position as a *Privatdozentin* will remain untouched; neither does this title entail the assignment of any officially authorized function". In other words, it was a "title without means" (*Titel ohne Mittel*); it was the least the Ministry could do for Emmy Noether.

(pp. 168–169) 'as a result of Courant's efforts she received a so-called *Lehrauftrag*, i.e., a small salary (200–400 marks per month) for her lectures, which required reconfirmation every year by the Ministry. It was in this position, without even a guaranteed salary, that she lived until the moment she was dismissed from the university and forced to leave Germany.'

'H. Weyl says this in the Memorial Address (pp. 127–128):"When I was called permanently to Göttingen in 1930, I earnestly tried to obtain from the Ministerium a better position for her, because I was ashamed to occupy such a preferred position beside her whom I knew to be my superior as a mathematician in many respects. I did not succeed, nor did an attempt to push through her election as a member of the Göttingen Gesellschaft der Wissenschaften. Tradition, prejudice, external considerations, weighted the balance against her scientific merits and scientific greatness, by that time denied by no one. In my Göttingen years, 1930–1933, she was without doubt the strongest center of mathematical activity there, considering both the fertility of her scientific research program and her influence upon a large circle of pupils."

Was Emmy Noether a 'blue stocking'?

(pp. 130–132) 'She lived in close communion with her pupils; she loved them, and took interest in their personal affairs. They formed a somewhat noisy and stormy family, "the Noether boys" as we called them in Göttingen. Among her pupils proper I may name Grete Hermann, Krull, Hölzer, Grell, Koethe, Deuring, Fitting, Witt, Tsen, Shoda, Levitzki. F. K. Schmidt is strongly influenced by her, chiefly through Krulls' mediation. V. d. Waerden came to her from Holland as a more or less finished mathematician and with ideas of his own; but he learned from Emmy Noether the apparatus of notions and the kind of thinking that permitted him to formulate his ideas and to solve his problems. Artin and Hasse stand beside her as two independent minds whose field of production touches on hers closely, though both have a stronger arithmetical texture. With Hasse above all she collaborated very closely during her last years. From different sides, Richard Brauer and she dealt with the profounder structural problems of algebras, she in a more abstract spirit, Brauer, educated in the school of the great algebraist I. Schur, more concretely operating with matrices and representations of groups; this, too, led to an extremely fertile cooperation. She held a rather close friendship with Alexandroff in Moscow, who came frequently as a guest to Göttingen. I believe that her mode of thinking has not been without influence upon Alexandroff's topological investigations. About 1930 she spent a semester in Moscow and there got into close touch with Pontrjagin also. Before that, in 1928–1929, she had lectured for one semester in Frankfurt while Siegel delivered a course of lectures as a visitor in Göttingen.

(p. 178) 'In her house—more precisely, in the mansard-roofed apartment she occupied in Göttingen (Friedländerweg 57)—a large group would get together eagerly and often. People of diverse scholarly reputations and positions—from Hilbert, Landau, Brauer and Weyl to the youngest students—would gather at her home and feel relaxed and unconstrained, as in few other scientific salons in Europe. These "festive evenings" in her apartment were arranged on any

> possible occasion; for example, in the summer of 1927 it was the frequent visits of her student van der Waerden from Holland. The evenings at Emmy Noether's, and the walks with her outside town, were a shining and unforgettable feature of the mathematical life of Göttingen for an entire decade (1923–1932). Many lively mathematical conversations were held during these evenings, but there was also much gaiety and laughter, good Rhine wine would sometimes be on the table and many sweets would be consumed . . .
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> Such was Emmy Noether, the greatest of women mathematicians, a leading scientist, wonderful teacher and unforgettable person. She did not have the characteristics of the so-called "women scholar" or "blue stocking". To be sure, Weyl said in his obituary, "No one could contend that the Graces had stood by her cradle," and he is right, if we have in mind her well-known heavy build. But at this point Weyl is speaking of her not only as a major scientist, but as a major woman! And this she was—her feminine psyche came through in the gentle and delicate lyricism that lay at the foundation of the wide-ranging but never superficial relationships connecting her with people, with her avocation, with the interests of all mankind. She loved people, science, life with all the warmth, all the joy, all the selflessness and all the tenderness of which a deeply feeling heart—and a woman's heart—was capable.

Is it true that the classical paper: Abstrakter Aufbau der Idealtheorie in algebraischen Zahl-und Funktionenkörpern, Math. Ann. **96** (1927), 26–61, containing the now famous five Dedekind domain axioms constituted the first 'abstract algebra' publication of the Noether School?

> (pp. 103–104) 'Through the study of the arithmetical theory of algebraic functions (14), Emmy Noether became familiar with Dedekind's theory of modules and ideals, which helped to determine the direction of her further work. In the paper she produced in collaboration with Schmeidler (17), concepts from the theory of modules—direct sums and intersections, residue class modules, isomorphy of modules—are developed and tried; they appear like a red thread throughout her later work. In this paper, also, uniqueness proofs are given for the first time by means of the method of exchange, and the representation of modules as intersections is achieved by means of a finiteness condition.
>
> The first major success of this method was achieved in 1921 in the paper, *Idealtheorie in Ringbereichen* (19), which has become a classic.'

In other words Emmy Noether's mathematical work, even the work by which contemporary mathematicians remember her best was far more 'universal' both by intent and by achievement than any 'specialist' is willing to concede. She did react creatively to existing mathematical theories, enriching them by the strength of her conceptual analysis, not merely doing them over according to a standard prescription.

Is Emmy Noether whose name and authority the categorists invoke as founding authority herself an abstract formalist? Herman Weyl says

> (pp. 140–141) 'In a conference on topology and abstract algebra as two ways of mathematical understanding, in 1931, I said this: Nevertheless I should not pass over in silence the fact that today the feeling among mathematicians is beginning to spread that the fertility of these abstracting methods is approaching exhaustion. The case is this: that all these nice general notions do not fall into our laps by themselves. But definite concrete problems were first conquered in their undivided complexity, single-handed by brute force, so to speak. Only afterwards the axiomaticians came along and stated: Instead of breaking in the door with all your might and bruising your hands, you should have constructed such and such a key of skill, and by it you would have been able to open the door quite smoothly. But they can construct the key only because they are able, after the breaking in was successful, to study the lock from within and without. Before you can generalize, formalize and axiomatize, there must be a mathematical substance. I think that the mathematical substance in the formalizing of which we have trained ourselves during the last decades, becomes gradually exhausted. And so I foresee that the generation now rising will have a hard time in mathematics.
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> Emmy Noether protested against that: and indeed she could point to the fact that just during the last years the axiomatic method had disclosed in her hands new, concrete, profound problems by the application of non-commutative algebra upon commutative fields and their number theory, and had shown the way to their solution.'

to which P. S. Alexandrov adds the following significant comment

> 'Much in this quotation deserves our attention. In the first place, one cannot, of course, dispute the point of view that every axiomatic presentation of a mathematical theory must be preceded by a concrete, one might say a naive mastery of it; that, moreover, axiomatics is only interesting when it relates to tangible mathematical knowledge (what Weyl calls "mathematical substance"), and is not tilting at windmills, so to speak. All of this is indisputable, and, of course, it was not

> against this that Emmy Noether protested. What she protested against was the pessimism that shows through the last words of the quotation from Weyl's speech of 1931; the substance of human knowledge, including mathematical knowledge, is inexhaustible, at least for the foreseeable future—this is what Emmy Noether firmly believed. The "substance *of the last decades*" may be exhausted, but not mathematical substance in general, which is connected by thousands of intricate threads with the reality of the external world and human existence. Emmy Noether deeply felt this connection between all great mathematics, even the most abstract and the real world; even if she did not think this through philosophically, she intuited it with all of her being as a great scientist and as a lively person who was not at all imprisoned in abstract schemes. For Emmy Noether mathematics was always knowledge of reality, and not a game of symbols; she protested fervently whenever the representatives of those areas of mathematics which are directly connected with applications wanted to appropriate for themselves the claim to tangible knowledge. In mathematics, as in knowledge of the world, both aspects are equally valuable: the accumulation of facts and concrete constructions and the establishment of general principles which overcome the isolation of each fact and bring the factual knowledge to a new stage of axiomatic understanding.'

One must be grateful to Auguste Dick, to her competent translator, to her many helpers (e.g. Olga Taussky) and to the publisher for submitting to the English reader a lively and well-researched report on the life and work of a mathematician whose scientific influence is with us every day but whose life had become a legend already in my student days.

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Product integration, by John D. Dollard and Charles N. Friedman, Encyclopedia of Mathematics and its Applications, Vol. 10, Addison-Wesley, Reading, Mass., 1979, xxii + 253 pp., $24.50.

The idea of product integration was first introduced by Volterra in his study of the evolution differential equation

$$dy/dt = A(t)y. \tag{1}$$

Let us consider the case where A maps an interval $[a, b]$ into the set of linear operators on a normed vector space $(X, \|\cdot\|)$ and y has the initial value