author does not make far-ranging conjectures, and does not philosophize. The book is lean and beautiful.

REFERENCES

1. B. J. Birch [**1957**], *Homogeneous forms of odd degree in a large number of variables*, Mathematika **4**, 102–105.

2. H. Davenport [**1962**], *Analytic methods for diophantine equations and diophantine inequalities*, Univ. of Michigan, Fall semester 1962, Campus Publishers.

3. H. Davenport [**1963**], *Cubic forms in* 16 *variables*, Proc. Roy. Soc. Ser. A **272**, 285–303.

4. H. Davenport and H. Heilbronn [**1946**], *On indefinite quadratic forms in five variables*, J. London Math. Soc. (2) **21**, 185–193.

5. H. Davenport and D. J. Lewis [**1969**], *Simultaneous equations of additive type*, Philos. Trans. Roy. Soc. London Ser. A **264**, 557–595.

6. M. J. Greenberg [**1969**], *Lectures on forms in many variables*, Benjamin, New York and Amsterdam.

7. G. H. Hardy and J. E. Littlewood [**1919**], *A new solution of Waring's problem*, Quart. J. Math. **48**, 272–293. (See also Hardy's collected papers, vol. I, Clarendon Press, Oxford, 1966, pp. 382–403.)

8. G. H. Hardy and S. Ramanujan [**1918**], *Asymptotic formulae in combinatory analysis*, Proc. London Math. Soc. (2) **17**, 75–115.

9. D. Hilbert [**1909**], *Beweis für die Darstellbarkeit der ganzen Zahlen durch eine feste Anzahl n-ter Potenzen* (Waringsches Problem), Göttinger Nachrichten, 17–36.

10. C. Hooley [**1981**], *On a new approach to various problems of Waring's type*, Recent Progress in Analytic Number Theory (Sympos., Durham, July 1979), Academic Press, New York, pp. 127–191.

11. L. K. Hua [**1938**], *On Waring's problem*, Quart. J. Math. **9**, 199–202.

12. J. I. Igusa [**1978**], *Lectures on forms of higher degree*, Tata Inst. Fundamental Research, Bombay.

13. Yu. V. Linnik [**1960**], *All large numbers are sums of a prime and two squares* (A problem of Hardy and Littlewood). I, Mat. Sb. (N.S.) **52** (94), 661–700. (Russian)

14. W. M. Schmidt [**1980**], *Diophantine inequalities for forms of odd degree*, Adv. in Math. **38**, 128–151.

15. ______ [to appear], *On cubic polynomials*. II–IV; Monatsh. Math. Part I **93** (1982), 63–74.

16. I. M. Vinogradov [**1928**], *Sur le théorème de Waring*, C. R. Acad. Sci. USSR, 393–400.

17. ______ [**1937**], *Representation of an odd number as a sum of three primes*, C. R. Acad. Sci. USSR **15**, 6–7.

18. ______ [**1947**], *The method of trigonometrical sums in the theory of numbers*, "Nauka" Interscience, New York.

19. ______ [**1971**], *The method of trigonometrical sums in the theory of numbers*, Moscow. (Russian)

20. H. Weyl [**1916**], *Über die Gleichverteilung von Zahlen mod Eins*, Math. Ann. **77**, 313–352.

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Group theoretic methods in bifurcation theory, by D. H. Sattinger, Lecture Notes in Math., vol. 762, Springer-Verlag, Berlin, Heidelberg, 1979, 241 pp., $14.00.

Analysis of nonlinear problems has always been a rich area in terms of mathematical difficulties and interesting problems. For the last two decades there has been a flurry of activity related to nonlinear eigenvalue problems and more specifically to bifurcation problems. During this period much progress has been made in bifurcation theory.

One basic approach that has evolved is to apply the Implicit Function Theorem (IFT) for values of the eigenparameter λ to obtain local uniqueness. This approach is continued until a nonempty null space of the Fréchet derivatives of the operator makes the IFT no longer applicable. The value of the eigenparameter at which this happens is a potential bifurcation point, and the method then is to project out the null space of the derivative of the nonlinear operator evaluated at the potential bifurcation point and again apply the IFT to prove the existence of a branch of nontrivial solutions. The stability of this new branch of solutions can then be investigated by once more using perturbation theory and the IFT. For an excellent discussion of the above approach, see [**1**].

The method of the preceding paragraph is essentially impossible when the dimension of the null space is large. Most often the dimension of the null space can be reduced by considering a restricted space of functions, and many problems have been solved by restricting the space to the extent that the null space is one dimensional. Of course, the resulting solution is not complete, the biggest shortfall being that the stability considered applies only to perturbations of functions in the restricted function space. No information is obtained concerning stability to more general perturbations. See [**2**] for a discussion of stability of bifurcating branches.

Most often the dimension of the null space is large due to a class of related symmetries. As Sattinger points out in [**3**], the fact that the equations of continuum mechanics are invariant with respect to a subgroup of the Euclidean group is a consequence of their derivation. Hence, such an invariance is often present.

These symmetries were recognized and handled by several authors in the late 1960's and early 1970's. See, for example, [**4** or **5**]. However, in an excellent series of papers Sattinger [**6**, **7**, **8**, **9**] began the work of formally developing bifurcation and stability theory of functions that are equivariant with respect to a given group.

Since the appearance of the papers by Sattinger the subject of group theoretic bifurcation theory has expanded greatly. This expansion includes applications such as [**10** and **11**], the book under review and the set of lecture notes [**3**]. In addition, similar results have been obtained via singularity theory in [**12**, **13** and **14**].

The book *Group theoretic methods in bifurcation theory* begins with an excellent review of bifurcation and stability theory and a chapter on some of the essentials of group representation theory which this reviewer felt was too brief. However, the major content of the book is a research monograph based largely on the papers [**6**, **7**, **8** and **9**]. The material presented has been superseded by the work of [**12**, **13**, **14** and **3**]. In fact, as is shown in both [**14** and **3**], the results of the main application given in the book, namely symmetry breaking in the Bénard problem, are incomplete.

The monograph does, however, still have its merits. The review of bifurcation theory and the discussion of the general setting for the application of group theoretic methods to bifurcation theory make it an excellent preliminary to the notes [3] or the papers [**12**, **13** and **14**]. The main shortcoming of the book is that, as perhaps should be the case with any three year old research monograph, it is no longer the latest word in the field.

REFERENCES

1. D. H. Sattinger, *Topics in stability and bifurcation theory*, Lecture Notes in Math., vol. 309, Springer-Verlag, Berlin and New York, 1973.

2. Klaus Kirchgässner, *Bifurcation theory in nonlinear hydrodynamic stability*, SIAM Review **17** (1975), 652–683.

3. D. H. Sattinger, *Bifurcation and symmetry breaking*, notes presented at the NSF-CBMS Conference, Univ. of Florida, 1981.

4. F. Busse, *The stability of finite amplitude cellular convection and its relation to an extremum principle*, J. Fluid Mech. **30** (1967).

5. Klaus Kirchgässner and Hansjörg Kielhöfer, *Stability and bifurcation in fluid dynamics*, Rocky Mountain J. Math. **3** (1973), 275–318.

6. D. H. Sattinger, *Selection mechanisms for pattern formations*, Arch. Rational Mech. Anal. **66** (1977), 31–42.

7. ______, *Group representation theory and branch points of nonlinear functional equations*, SIAM J. Math. Anal. **8** (1977), 179–201.

8. ______, *Bifurcation from rotationally invariant states*, J. Math. Phys. **19** (1978).

9. ______, *Group representation theory, bifurcation theory and pattern formation*, J. Funct. Anal. **28** (1978), 1720–1732.

10. George H. Knightly and D. Sather, *Applications of group representations to the buckling of spherical shells*, Applications of Non-linear Analysis in the Physical Sciences (H. Amann et al (eds.)), Pitman, London, 1981.

11. ______, *Regularity and symmetry properties of solutions of the John Shell equations for a spherical shell*, Contemporary Math., vol. 4, Amer. Math. Soc., Providence, R. I., 1981, pp. 45–59.

12. M. Golubitsky and D. Schaeffer, *A theory for imperfect bifurcation via singularity theory*, Comm. Pure Appl. Math. **32** (1979), 21–98.

13. ______, *Bifurcation with* $O(3)$ *symmetry including applications to the Bénard problem*, Comm. Pure Appl. Math. (to appear).

14. Ernesto Buzano and M. Golubitsky, *Bifurcation involving the hexagonal lattice*, Technical Report #56, Department of Mathematics, Arizona State University, 1981.

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