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Locally convex spaces, by H. Jarchow, Teubner, Stuttgart, 1981, 14 + 534 pp., DM 98,--.

Modern methods in topological vector spaces, by Albert Wilansky, McGraw-Hill, 1979, xiii + 298 pp., $39.95.

In the preface to the 1973 English translation [4] of his 1954 notes on locally convex spaces, Grothendieck wrote to the effect that the translation was verbatim and that no attempt had been made to update the notes since *nothing had happened in the theory of locally convex spaces* since the appearance of his notes twenty years earlier.

In his 1976 review of the translation, John Horváth wrote [8] "Even after twenty years, Grothendieck's book is an elegant and refreshing introduction to topological vector spaces, and in spite of the fact that at least *ten* monographs have been written on the subject since 1954, it is probably the best text book to use in a course."

Horváth's own book on the subject [9] appeared in 1966!

Continuing with Horváth on Grothendieck: "The proofs are at times concise or even omitted, but this enhances its value as a text book. An additional feature is the 195 exercises, some of them quite challenging, which cover about 61 of the books' 245 pages."

Good news you Bornovores! Roll out the Barrels! Two more books have entered the TVS sweepstakes. They are Ptaking over. (Every reviewer is allowed one atrocious pun.)

For conciseness (read terseness) Wilansky makes Grothendieck read like William Faulkner. Not since mimeographed sheets (à la R. L. Moore) passed out in my first topology class, have I seen definitions, theorems, propositions, lemmas, examples, and corollaries spouted out so rapidly. Chapter 3 of Wilansky, modestly called "Banach spaces," is ten pages long (two of these pages devoted to exercises). Did I say exercises? Wilansky has 1500 of them. Alas, Jarchow has none! But remember, Grothendieck could only come up with 195!

Before discussing the similarities and differences of Wilansky (W) and Jarchow (J) let us consider the audience these books are purportedly addressing.

Jarchow states as prerequisites general topology and measure theory but (carefully) does not define the level of graduate work for which his book is intended. The blurb to Wilansky's book, on the other hand, asserts that the book "is well suited to the needs of undergraduate and graduate students, since the material and problems are arranged so as to be self-contained." Perhaps there is some black humor intended here. Consider the following "proof" of the Radon-Nikodym theorem on p. 239 (W): "See [**3**, Theorem III.10.7]; or [**7**, Theorem 31B]. For the last part see [**16**, Theorem 6.12]." Or problem 301, p. 66: "(P. Enflo). There exist a separable Banach space with no (Schauder) basis. [See [**13**, pp. 12 and 48].]" (I changed the numbers of the references from those in W to those listed at the end of this review.)

Of course, the latter is Enflo's negative solution to the approximation problem, and I have no qualms about including such material in this underhanded way. I do quibble with "self-contained" however. Wilansky for undergraduates or first year graduate students? I doubt it. Most students at this level could probably find the library, but, I question their ability to make the above (and many, many, more) examples viable. Moreover, according to Wilansky, "a few problems whose numbers are below 100 are considered part of the text." By my count, there are 300 such problems!

Now, the comparisons.

$W \cap J$. Both books contain the material I would consider the standard topics for a course in locally convex spaces: Duality, seminorms, completeness, compactness, open maps and closed graph theorems, barrels, bornologies, equicontinuity, reflexivity, etc. Also the major theorems: Hahn-Banach, Banach-Steinhaus, Krein-Milman, Mackey-Arens, etc. Indeed, except for some slight differences of presentation, any of the existing books on the subject (e.g. [**10**, **11** or **18**]) would have the same material.

$W \cup J$. Together the books contain more than anyone would want to know about locally convex spaces unless actively engaged in research in the area.

W $\setminus$ J. The main feature here is "the equivalence program." Consider e.g. the theorem: *X is barrelled if and only if weak* bounded sets in X′ are equicontinuous*. This statement is of the form $P \equiv (Q \to R)$ with P a property of X and Q, R properties of X'. This equivalence program is exploited extensively in the last half of W.

W also devotes a lot of attention to the separable quotient problem for Banach spaces: Does every Banach space have a separable quotient? This simply stated problem is still unsolved! To the delight of the locally convex spacers, the problem is equivalent to the following: Every Banach space X has a dense nonbarrelled subspace D.

The best features of W are the extensive tables at the end of the book. These thirty-three tables neatly summarize the material in the book and are a valuable aid to understanding the various properties of locally convex spaces. For example if one wonders if the completion of a bornological space is necessarily bornological, a brief look at table 12 shows that the answer is no.

There is one puzzling slip that needs mention. On p. 146 W proves Sobczyk's thoerem: *Let X be a separable normed space containing c_0 (the space of null sequences with the sup norm). Then there is a projection P: $X \to c_0$ with* $\|P\| \leq 2$.

He then states that it is unknown whether c_0 is the only Banach space with this property. But, it *is* known. Zippin [**20**] proved this in 1977. There are references in W as late as 1978. It is also strange that none of Grothendieck's work on locally convex spaces is referenced.

A word of warning: In W, "Fréchet" means "complete metric linear space." While historically I believe this to be accurate, the term now almost universally means "complete locally convex metric linear space" (a B_0-space in the earlier Polish terminology). A word of thanks: Thank you W for dropping "Balloon" [**19**].

J $\setminus$ W. In the theory of locally convex spaces J devotes considerable attention to webbed spaces and the very general closed graph theorem of DeWilde [**2**]. There is also much material on Schauder bases in locally convex spaces. This material recalls the sins of youth!

Most importantly J presents (in great detail) material on tensor products and nuclear spaces. This material isn't even mentioned in W. This is surprising, since the nuclear spaces are undoubtedly the most important nonnormable locally convex spaces.

The material on tensor products closely follows the fundamental work of Grothendieck [**5, 6**]. The proof of the Grothendieck inequality (called by Grothendieck the fundamental theorem in the metric theory of tensor products) is taken from [**12**].

This material is used in the ideal theory of operators on Banach spaces. While the tensor product theory can be found in [**5, 6**], the nuclear space theory in [**5, 15**], and the ideal theory in [**14**], the book of J presents the essentials in one place. Moreover J accomplishes this in 196 pages. Another nice feature of J is the "References" section at the end of the chapters. Following [**3**] (I guess) this makes the extensive bibliography more meaningful.

Finally the (gDf)-spaces of Ruess [17] are presented. This seems the "natural" setting for many of Grothendieck's compactness results and for applications to continuous function spaces.

It is also nice to see the Davis-Figiel-Johnson-Pełczyński factorization theorem (a weakly compact map factors through a reflexive space) [1] makes its first appearance in a book. Interestingly, W is a reference in J.

Conclusions. These brief comparisons in no way adequately cover the material presented in these two books. Both are well written, carefully thought out, and would both make excellent text books (at some level). I like J better (probably because the third part of the book is closest to my own interest). Moreover, I really find it hard to believe that W is aiming for a first year audience or that he really expects many of the 1500 exercises to be worked. Modulo this, the book is very readable and probably teachable.

However, I suspect that the biggest demand for either book will be for reference purposes. As already mentioned, the tables of W are superb and part III of J is indispensable to anyone not owning the references alluded to above.

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