out what he is doing but also says why he is doing it. The exercises are usually interesting but sometimes routine.

This book is accessible to all mathematicians and any graduate student who has had a course in abstract algebra. For the latter, some of the references (eg. references to algebraic varieties) will be obscure, but not to the point of making the book unreadable. Furthermore, the book presents clearly many of the standard techniques of abstract algebra in a fashion which is helpful to a second year graduate student. The exposition is dry but clear, so that the book could be used in a reading course. It is not a book to read quickly to get the flavor of K-theory, but rather a book to be worked through to gain a feel for the subject.

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Emmy Noether, a tribute to her life and work, edited by James W. Brewer and Martha K. Smith, Pure and Applied Mathematics: A Series of Textbooks and Monographs, vol. 69, Dekker, New York and Basel, 1981, x + 180 pp., $14.75.

The centenary of E. Noether's birth gave rise to two English language tributes very different in intent, background and execution. The first one was written in German in 1968; it was translated in English only in 1980. Its author, a teacher of mathematics living in Vienna, was motivated by a lecture of Professor E. Hlawka (Vienna University) on the development of mathematics in the last hundred years, as she states in the preface of her book. She traced very carefully the available documentary evidence of the life path and the scholarly development of E. Noether in context (see review of *Emmy Noether*, 1882–1935, by Auguste Dick, in Bull. Amer. Math. Soc. **6** (1982), 224–230). The second one is specially written for the occasion by mathematicians who encountered and applied Noetherian modes of thought in their work. The desire to describe the influence of E. Noether on contemporary research brought forth a number of articles (see 6–10 of the book under review) containing semipopular expositions of *Noether's mathematics* by five specialists (R. G. Swan, E. J. McShane, R. Gilmer, T. Y. Lam, A. Fröhlich). A biography entitled: *Emmy Noether and her influence*, by Clark Kimberling (pp. 1–64) followed by personal recollections of Saunders Mac Lane (pp. 65–78) and Olga Taussky (pp. 79–92) relating to E. Noether and new translations of the obituaries of B. L. van der Waerden and P. S. Alexandroff serve as introduction under the headings: BIOGRAPHY, NOETHER AND HER COLLEAGUES.

As an appendix we find a translation of NOETHER'S ADDRESS TO THE 1932 INTERNATIONAL CONGRESS OF MATHEMATICIANS (*Hypercomplex systems and their relations to commutative algebra and number theory*) as well as a publication list of Emmy Noether, and the Index.

On the whole the biographic article by Kimberling is eclectic, piecing together information already contained in A. Dick's book and other biographic material without further elucidation. However, there are some valuable quotations supplementing the biographic account given by A. Dick. From a letter by A. Einstein to Hilbert dated May 24, 1918 (p. 13) we obtain the first reaction of a theoretical physicist to *Noether's mathematics*. In the same vein is a quotation from a letter of Peter G. Bergmann written to the author on September 13, 1968. On p. 23 there is an interesting quotation from Alexandroff 1969. More detail is provided relating to E. Noether's appointment as Professor at Bryn Mawr (1933–1935) setting in sharp focus the spirit of hospitality and enterprise of her new country.

This tribute to E. Noether contains 10 more photos of E. Noether, her father and her colleagues supplementing those in A. Dick's book. However, most interesting for the mathematician is the postcard to Professor E. Fischer of April 10, 1915 (following p. 58 of A. Dick's book), a typical example of E. Noether's mode of presentation.

One expects a new translation of B. L. van der Waerden's obituary to be an improvement over H. I. Blocher's translation contained in A. Dick's book. This one is not. Referee found upon close study more than 20 smaller and larger inaccuracies, the worst one is the following passage (p. 93): 'Her absolute, incomparable uniqueness cannot be explained by her outward appearance only, however characteristic this undoubtedly was. Her individuality is also by no means exclusively a consequence of the fact that she was an extremely talented mathematician, but lies in the whole structure of her creative personality, in the style of her thoughts, and the goal of her will' which is offered as a translation of B. L. van der Waerden's: 'Ihre absolute, sich jedem Vergleich entziehende Einzigartigkeit ist nicht in der Art ihres Auftretens nach aussen hin zu erfassen, so charakteristisch dieses zweifellos war. Ihre Eigenart erschöpft sich auch keines wegs darin, dass es sich hier um eine *Frau* handelt, die zugleich eine hochbegabte Mathematikerin war, sondern liegt in der ganzen Struktur dieser schöpferischen Persönlichkeit, in dem Stil ihres Denkens und dem Ziel ihres Wollens.' Compare the new translation with H. J. Blocher's translation: 'Her originality, absolute beyond comparison, was not a matter of her bearing, characteristic though this was. Nor did it exhaust itself in the fact that this highly gifted mathematician was a woman. Rather it lay in the fundamental structure of her creative mind, in the mode of her thinking, and in the aim of her endeavours.' What's the use of 'avoidance of Germanisms' if the characteristic terseness of van der Waerden's German style (often used by my colleagues for the purpose of German language tests) gets slaughtered in the process of translation and if the content of his oration, the mature fruit of 10 years of collaboration with a friend and colleague, is garbled beyond recognition. The first duty of the translator of foreign prose is to hit squarely at the meaning of what the speaker had to say. Secondly, to convey an inkling of his manner of speaking to the foreign reader.

The readers of this particular translation will never know what they are missing unless they consult the original or the book of A. Dick.

Blocher's translation comes much closer to the mark, in general. The reader will have no difficulty recognizing van der Waerden's characteristic way of expressing himself in German (more clearly than most German born mathematicians), the meaning of his words is there though perhaps with a Germanic slant of phrasing.

No doubt, a perfect rendering may be unattainable. For example the famous passage in which B. L. van der Waerden formulates the maxim by which Emmy Noether was guided throughout her work: "Alle Beziehungen zwischen Zahlen, Funktionen und Operationen werden erst dann durchsichtig, verallgemeinerungsfähig und wirklich fruchtbar, wenn sie von ihren besonderen Objekten losgelöst und auf allgemeine begriffliche Zusammenhänge zurückgeführt sind." is neither fully rendered by Blocher's: "Any relationships between numbers, functions and operations only become transparent, generally applicable, and fully productive after they have been isolated from their particular objects and been formulated as universally valid concepts" nor by "All relations between numbers, functions and operations become clear, generalizable, and truly fruitful only when they are separated from their particular objects and reduced to general concepts" when the climax of the maxim as given in the German original really says more: "...only when they are separated from their particular objects and reduced to the terms of a general conceptual context". In other words, a mathematician who wants to breathe the spirit of E. Noether and her school is well advised to learn German. And, of course, he or she should not be content to accept van der Waerden's enthusiastic oration as the only information on the energizing influence of E. Noether's personality. E.g., van der Waerden's characterization (A. Dick, p. 101): 'She was unable to grasp any theorem, any argument unless it had been made abstract and thus made transparent to the eye of her mind. She could only think in concepts, not in formulas, and precisely here lay her strength. It was the very nature of her mind which compelled her to invent conceptual forms which were suitable as carriers for mathematical theories.' stands in stark contrast against the background of facts as we know them both from E. Noether's published work and from the published biographies (e.g. A. Dick's). She was quite well able to grasp both theorems and arguments in their concrete form, as her thesis shows as well as the introductory remarks of many later publications, and as is confirmed by contemporary witnesses. Fact is: *She did not want to get hold of a theorem or an argument unless it had been made transparent to the eye of her mind.*

Indeed, it would be sad for all of us who emulate E. Noether's maxim in our work if she would have been endowed with a particular gift making her mind different in essence and performance from ours. She was the pioneer who by her indomitable conquering spirit taught us not to be afraid of 'begriffliches Denken' in mathematics.

Also P. S. Alexandroff's 1935 memorial address was translated anew. This translation is perhaps more idiomatic than Blocher's (A. Dick, pp. 153–179) in some places, on the whole the two translations are equivalent.

The valuable article on *Galois theory* by Richard G. Swan gives a full account of the current 'state of the art' of algebraically solving the inverse problem of Galois theory over a given field of reference in the light of E. Noether's fundamental paper: Gleichungen mit vorgeschriebener Gruppe, Math. Annalen **78** (1918), 221–229. Corig., Math. Annalen **81**, 30.

Article 7 by E. J. McShane on *The calculus of variations* reproduces the bare mathematical skeleton of E. Noether's famous article: *Invariante Variationsprobleme* (Nachr. Ad. König. Gesellsch. d. Wiss. zu Göttingen, Math.-phys. Klasse (1918), 235–257) providing intrinsic evidence of today's estrangement of the pure mathematics establishment from the source of interesting problems. How come that E. Noether's work had enormous influence on several generations of mathematical physicists?

The article 8 on *Commutative ring theory* by Robert Gilmer deals with E. Noether's influence on the development of modern ring theory mainly from the historical viewpoint though not penetrating to the historical root of her work. What was so great about Dedekind's influence on E. Noether's axiomatic approach?

On the other hand the article 9 on *Representation theory* by T. Y. Lam is a very worthwhile introduction to representation theory for the modern student developing a vivid picture how the subject first was created in the twenties. It also draws well the connecting lines from earlier work of Cartan, Molien and McLagan-Wedderburn to E. Noether's unified theory making ample use of T. Hawkins' study.

Finally the article 10 on *Algebraic number theory* by A. Fröhlich sums up brilliantly on 7 pages E. Noether's achievements relating to the Galois module structure of algebraic number fields and their influence on modern research.

I guess Emmy Noether would be very pleased with the influence she has had on subsequent generations of mathematicians after her early passing from the scene; there are quite a few 'Noether boys' around today that she has neither seen nor heard of. Perhaps the best measure of her lasting influence is the very fact that the editors of the book on Emmy Noether arranged as *A tribute to her life and work* did not even attempt to have categorized, analyzed and assessed her 'total' achievement in her chosen field of endeavour. Instead we see five specialists dealing with particular aspects of her work which they discovered to be of basic importance in their own pursuits.

As Olga Taussky mentions in *My personal recollections of Emmy Noether*: 'Fröhlich's contribution to this book deals with Emmy's influence on algebraic number theory—class field theory, with cohomology playing a vital role. It seems certain that she could have done more there. It is futile to wonder what it might have been.'

Like C. F. Gauss, the fountainhead of Göttingen's influence on the mathematical world, Emmy Noether also had an inner vision of mathematical truth which impelled her to go beyond the confines of the specialist's work. In many respects her inner vision complements that of C. F. Gauss. But, where it takes arduous studies and a life time of commitment to discover C. F. Gauss's program behind the cool marble of his finished works, Emmy Noether communicated her ideas in her active years freely and convincingly to many people and through them to subsequent generations of scholars. It is safe to predict that many more (and perhaps even more inspired) tributes to her life and work are going to appear in the future.

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An introduction to variational inequalities and their applications, by David Kinderlehrer and Guido Stampacchia, Academic Press, New York, 1980, xiv + 313 pp., $35.00.

The theory of variational inequalities (= V.I.) was born in Italy in the early sixties. The "founding fathers" were G. Stampacchia and G. Fichera. Stampacchia was motivated by potential theory, while Fichera was motivated by mechanics (problems in elasticity with unilateral constraints; see III.2). Less than twenty years later the theory of V.I. has become a rich source of inspiration *both* in pure and applied mathematics. On the one hand, V.I. have stimulated new and deep results dealing with nonlinear partial differential equations. On the other hand, V.I. have been used in a *large variety* of questions in mechanics, physics, optimization and control, linear programming, engineering, etc...; today V.I. are considered as an indispensable tool in various sectors of applied mathematics.

I. What is a V.I.? V.I. appears in a natural way in the *calculus of variations* when a function is minimized over a convex set of constraints. In this case the classical Euler equation must be replaced by a set of inequalities. Let us consider first a very simple example.

I.1. *V.I. in finite-dimensional spaces*. Let F denote a C^1 real valued function on $\mathbf{R}^n$ and let $K \subset \mathbf{R}^n$ be a closed convex set. If there is some $u \in K$ such that

$$F(u) = \operatorname*{Min}_{v \in K} F(v) \tag{1}$$

then u satisfies

$$\begin{cases} u \in K, \\ (F'(u), v - u) \geqslant 0 \quad \text{for all } v \in K. \end{cases} \tag{2}$$

In general a solution of (2) is not a solution of (1), unless F is convex.

EXAMPLE. $F(v) = |v - a|^2$ ($a \in \mathbf{R}^n$), then (2) reduces to the well-known characterization of the projection of a on K.