groups, incorporating a number of his own refinements and simplifications. The finer points will not be of interest to all readers, but the main line of development should appeal to anyone who is curious about what lies beyond Tannaka duality.

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Convex sets and their applications, by Steven R. Lay, Wiley, New York, 1982, xvi + 244 pp., $29.50.

Steven Lay teaches at an undergraduate institution and he wrote this book with his students in mind. Because of its title, the book invites comparison with the well-known book *Convex sets* by Lay's teacher, F. A. Valentine [**9**]. But Lay's intended audience calls for a different kind of book. He works not in a linear topological space, but in R^n. He motivates and clarifies his material with numerous diagrams and an occasional apt analogy. He follows each section with a carefully graded set of problems. And, as the title implies, he offers some applications; perhaps the best example is a chapter on optimization. In summary, Lay aims to do for convex sets what the authors of this review tried to do for convex functions [**7**].

Lay says in his preface that "there is no text at this level which has convex sets and their applications as its unifying theme"—a bit of an overstatement, we think. There are two fine books by Russian authors: *Convex figures* by Yaglom and Boltyanskii [**10**] and *Convex figures and polyhedra* by Lyusternik [**9**], though it could be argued that they are not textbooks in the American tradition. Benson's *Euclidean geometry and convexity* [**1**] is definitely a textbook but is is oriented toward plane and solid geometry. Kelly and Weiss cover much the same ground as Lay in *Geometry and convexity* [**5**] but their book is more topological and probably more difficult for undergraduates. And of course, there is a host of advanced books, of which Grünbaum's *Convex polytopes* [**3**], Eggleston's *Convexity* [**2**], Rockafellar's *Convex analysis* [**8**] and the aforementioned book of Valentine are worthy of note. There are, then, other books that are developed around the theorem of convex sets. But all of them can be clearly distinguished from Lay's book, which will thus be a useful addition to the literature.

Has Lay achieved his announced goal—to introduce undergraduates to "the broad scope of convexity"? Our answer is yes. The principal definitions and theorems are here, called by their familiar names, and nicely indexed. (We were surprised to discover that Valentine's index does not mention either Caratheodory's or Kirchberger's theorem.) The proofs are carefully arranged and readable. Most topics are pursued to modest depth, occasionally to the statement of an unsolved problem, then terminated with references to the literature. For example, Lay gives a nice treatment of polytopes, then appropriately steers the reader to Grünbaum's book for more information. Having recently looked for a readily accessible, direct, and reasonably complete discussion of barycentric coordinates before this book crossed our desk, we were pleased to find just such a presentation in Lay's book. And it is hard to imagine how anyone could improve on the clarity with which Lay presents Helly's Theorem in §6. Finally, we welcomed a number of references to the history of convex sets.

Lay's writing is clear and direct, though its Definition, Theorem, Corollary style is perhaps too formal for a book of this type. It makes the terrain being covered seem uniformly flat, when the intended audience needs to have pointed out those results that stand out as mountain peaks of achievement.

The coverage is understandably uneven. In the discussion of his specialty, Kirchberger's Theorem, Lay carries the reader to the edge of current research in considering separation by various geometrical objects, not just hyperplanes. On the other hand, since he briefly discusses curves of constant width, Lay could have performed a real service by collecting the widely scattered material on this subject, perhaps concluding with the beautiful theorem of Hammer and Sobczyk [**4**] which tells how to construct all such curves.

We noted a few misprints of the trivial variety. For example, the first sentence of the proof of Theorem 25.7 (Fundamental Theorem of Matrix Games) reads "Since $v(x)$ is continuous on the compact set X, Theorem 1.21 implies...". Replace 1.21 by 1.23 but, more importantly, why is $v(x)$ continuous? This key point in the proof of both this theorem and the Minimax Theorem seems to be glossed over.

These quibbles are minor, however, and do not obscure the overall attractiveness of the book. Ultimately we expect, the degree to which the goals of the book are realized depends upon the problem sets. Though we have not yet had opportunity to teach from the text, our perusal suggests that the problem sets will make study of this text a very worthwhile experience for any undergraduate.

Exercises in the problem sets are divided into three categories. (Lay's students refer to them as the easy, the hard, and the impossible.) The unmarked exercises illustrate and expand the text discussion in a straightforward manner. The starred exercises go beyond the scope of the text. Those marked with daggers present open-ended and unsolved problems. There are solutions, hints, and references for selected problems at the end of the book.

While on the subject of exercises, we'll point out that some will applaud, others will dislike, Lay's tendency to relegate parts of his proofs to the problem sets. That same statement could be made, of course, about the entire book. We are among those who are applauding.

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Counterexamples in topological vector spaces, by S. M. Khaleelulla, Lecture Notes in Math., vol. 936, Springer-Verlag, Berlin and New York, 1982, xxi + 179 pp., $10.70.

Here is a rule-of-thumb test to identity latent mathematicians: Make an assertion. If the young person tries to prove it, (s)he fails the test; if (s)he tries to find a counterexample, you have a future mathematician on your hands.

Examples are more important than theorems. If you teach me the rules of a game and attempt to develop a theory, I will interrupt to say "Let's play it once". A course in groups containing pure theory would allow the conjecture "All groups are commutative" to stand unchallenged—besides failing to educate the students.

The role of examples is educational: the derivative of a specific function, a group with 5 members; but we shall be concerned with those which are always thought of as counter: a nowhere differentiable function, a nonmeasurable set.

Is the earliest known counterexample the book of Job? (Assertion: Holiness brings good fortune.)

What is the role of counterexamples in mathematics? (Are there any in Euclid?) I attempt to list the roles in decreasing order of importance; the "big" examples fall early in my list:

1. To refute widely held beliefs. (A nowhere differentiable continuous function, a series whose sum is discontinuous.)

2. To show the need to work in a more general setting. (A nonsequential limit point.)

3. To show the inadequacy of a definition. (Space-filling curve: what does dimension mean?).