The monograph is well written. Despite some omissions, the book, which is intended for a broader audience, can also be an excellent reference book for a mathematican interested in nonlinear functional analysis. The bibliography of 39 pages is very impressive.

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MIECZYSLAW ALTMAN

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Groups, trees and projective modules, by Warren Dicks, Lecture Notes in Math., vol. 790, Springer-Verlag, Berlin and New York, 1980, 126 pp., $9.80.

Trees, by Jean-Pierre Serre, Springer-Verlag, Berlin and New York, 1980, ix + 142 pp., $29.80.

Classically, one studies a discrete subgroup Γ of a Lie group G by its action on the homogeneous space $X = G/K$ where K is a maximal compact subgroup of G; for torsion-free Γ, the form $\Gamma\backslash X$ is a $K(\Gamma, 1)$ space. When $G = SL_2(\mathbf{R})$ and $\Gamma = SL_2(\mathbf{Z})$ one has the well-studied reduction theory for Γ and its subgroups acting on the upper half-plane $\mathcal{H} = SL_2(\mathbf{R})/SO_2$ by linear fractional transformations. A program initiated by Bruhat and Tits makes available certain simplicial complexes called buildings which play the role of the symmetric space for p-adic groups [**20, 22, 28**]. For $G = SL_n(\mathbf{Q}_p)$ the building is a contractible $n - 1$ dimensional complex, $T_n(\mathbf{Q}_p)$, in which the vertices are the elements of $SL_n(\mathbf{Q}_p)/SL_n(\mathbf{Z}_p)$ and the simplices come from flags of $\mathbf{Z}_p$-submodules of $\mathbf{Q}_p^n$ which "cover" flags of subspaces of $(\mathbf{Z}/p\mathbf{Z})^n$. When $n = 2$ this Bruhat-Tits tree provides the background fiber to the first part of

Serre's 1968/69 course on discrete groups **[21]**. In fact, it was Serre's observation that discrete torsion-free subgroups of $SL_2(\mathbf{Q}_p)$ act freely on the Bruhat-Tits tree (thus reproving a theorem of Ihara that such groups are free) which prompted him to develop a general theory of groups acting on trees. As a result of Whitehead shows, the underlying motif of the universal covering of a $K(\Gamma, 1)$ space is a tree, where Γ is the free product with amalgamation, $A *_C B$. One then has the foundation for a revitalization of combinatorial group theory with the tools and techniques of the theory of groups acting on trees. In the second part of Serre's course **[20]** one finds the symmetric spaces and buildings used in combination to study a large class of groups and their cohomology. For example, $\Gamma = SL_2(\mathbf{Z}[1/p])$ is a discrete subgroup of $G = SL_2(\mathbf{R}) \times SL_2(\mathbf{Q}_p)$ and acts properly discontinuously on $X = \mathcal{H} \times T_2(\mathbf{Q}_p)$; since the quotient $\Gamma \backslash X$ has finite volume beautiful relationships result between the Euler characteristic of Γ and the Euler-Poincaré measure of $\Gamma \backslash G$ (and consequently certain zeta functions).

Serre's book on trees is the exposition of the first part of that course. It begins *ab initio* with a leisurely study of the two main constructions of combinatorial group theory: free products with amalgamation and the HNN extension. These arise in topology, for example, in the description of the fundamental group of a 3-manifold which splits along a connected incompressible surface. Furthermore, in each of these two group-theoretic constructions there is a naturally defined tree, T, on which the group acts so that the quotient by the group action is an edge or a loop, respectively. In the topological situation this is just imitating the action of the fundamental group on the universal covering space. Now, a subgroup Γ of an amalgam or HNN extension is not necessarily of such a simple type; but it does have an action, by restriction, on the associated tree T. One is thus led naturally to consider the more complex situation of a graph of groups on $Y = \Gamma \backslash T$ for which the amalgam and HNN (edge and loop) are the building blocks. By a graph of groups $(\mathcal{G}, Y)$ is meant an assignment of groups $v \to G_v$, $e \to G_e$ to the vertices and edges of a graph Y subject to certain compatibility conditions. From this one determines the fundamental group, $\Gamma = \pi(\mathcal{G}, Y)$, of the graph of groups, and a tree T with an action of Γ so that $\Gamma \backslash T = Y$ with the stabilizers of vertices and edges providing the necessary "assignment" of groups. Serre's *Trees* and Dicks' *Groups*, *trees* and *projective modules* differ slightly in the discussion of the details of the theory of graphs of groups. The latter emphasizes certain derivations which arise in the formation of the graph of groups from the quotient of the tree. Others, also, have redone this section of the theory according to their own favorite point of view **[7, 14, 18 30]**. Also, Lyndon's reworking of Nielsen's proof of the subgroup theorem for free groups **[16]** led him to introduce certain abstract length functions on groups which formalized the cancellation arguments of Nielsen. Chiswell **[6]** has shown that an integer-valued length function on a group Γ gives rise to a tree T and an action of Γ on T which provides an equivalent form of the Bass-Serre (cf. **[21**, p. 4**]**) graph of groups viewpoint. As Lyndon remarks, the values of these length functions need not be so restricted to obtain an interesting theory. There is now a "generalized tree" **[29]** for the action of a group with a real-valued length function [**2**]. Recently generalized trees have played a crucial role in work of Shalen and Morgan in analyzing the "Thurston compactification" of Teichmüller space and other aspects of the uniformization theorems for 3-manifold groups [**23**].

In the second chapter of *Trees*, Serre constructs the Bruhat-Tits trees. The exposition is lucid and complete with many interesting examples and applications. With this background one can then further appreciate the section on fixed points. The theory of group actions on trees is of course only useful in decomposing a group Γ into simpler pieces if the action of Γ is without a common fixed point. However, one would like to know the extent of the theory; thus, property FA occurs to describe the class of groups for which any action on a tree has a fixed vertex [**3**]. Serre shows that $SL_3(\mathbf{Z})$ has property FA and remarks on the extensions of this result by Tits and Margulis [**27**]. Recently, Watatani has shown that for countable groups, Kazhdan's property T implies property FA [**31**]; similar relationships persist also for p-adic groups so that, for example, $SL_3(\mathbf{Q}_p)$ has no nontrivial action on a tree [**1**]. In another interesting application of Bruhat-Tits trees, Serre shows that a countable group with property FA has only a restricted class of two-dimensional representations. Bass has reformulated and extended this result to give a characterization of subgroups of $GL_2(\mathbf{C})$, which has played an essential role in the resolution of the Smith Conjecture [**4**].

The second half of Dicks' book is concerned with the recent extensions of Stallings' theorem on groups with infinitely many ends and its consequences for groups of cohomological dimension one [**24**]. The treatment here follows Dunwoody's construction of the tree for an infinitely ended group which is based on the existence of certain cut-sets [**10, 24**]. The connections with groups of cohomological dimension one now appears via the derivations which one recovers from the action of the group on Dunwoody's tree and which in turn determine projective properties for the augmentation ideal. Indeed, the augmentation ideal of the group ring $R\Gamma$ is $R\Gamma$-projective if and only if Γ acts on a tree so that the stabilizers of vertices are finite groups whose order is invertible in the ring R. Recently, Dunwoody has shown that a countable group Γ has more than one end if and only if it is the fundamental group of a graph of groups with proper vertex subgroups and finite edge groups [**9**]. Also, Müller [**7**] has now given generalizations of the Swan and Swarup techniques for simultaneous decompositions for groups and a family of subgroups [**25, 26**]. This has now led to the complete characterization by Eckmann-Müller and Linnel of groups with cohomological dimension two and satisfying (a formal version of) Poincaré duality; they are precisely the surface groups [**11, 15**].

In the final sections of *Trees*, Serre studies the group $GL_2(K)$ where K is the function field of a smooth projective curve and its subgroup $GL_2(A)$ where A is the coordinate ring of an affine part of the curve with a unique point at ∞. This point gives rise to a discrete valuation v on K and hence the action of $GL_2(K)$ on the Bruhat-Tits tree $T_2(K_v)$. In this situation, one can give a reinterpretation of the vertices of this tree as vector bundles of rank two (modulo a homothety) over the curve which are trivial on the affine part. This point of view was extremely useful in Quillen's proof of the finite generation of the algebraic K-groups of a curve over a finite field [**13**]. Indeed, Quillen generalized Serre's homology calculations to show that the homology groups of $\mathrm{Aut}(P)$ are finitely generated where P is an arbitrary (finitely generated) projective module over A and the coefficients of the homologies lie in the Steinberg representation.

There have been numerous other applications of the Bruhat-Tits trees to harmonic analysis by Cartier [**5**], the congruence subgroup problem for SL_2 by Serre [**19, 21**], p-adic curves and Schottky groups by Mumford and others [**12**], and 3-dimensional topology by Culler and Shalen [**8**].

Although, in translation, *Trees* has lost its magnificent frontispiece and virtually tripled in price, it's still a great buy. These two volumes, in combination, are an excellent introduction to the theory of groups acting on trees and its various applications.

In the fourth paragraph, p. 96, of *Trees*, "principal ideals" should be "prime ideals."

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ROGER C. ALPERIN