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CR submanifolds of Kaehlerian and Sasakian manifolds, by Kentaro Yano and Masahiro Kon, Progress in Mathematics, Vol. 30, Birkhauser, Cambridge, Mass., 1982, x + 208 pp., $17.50. ISBN 3-7643-3119-4

The theory of submanifolds of Kaehlerian manifolds is one of the important branches of differential geometry. It began as a separate area of study in the 19th century with the investigation of projective varieties in a complex projective m-space $\mathbf{C}P^m$. It was J. A. Schouten and D. van Dantzig [**10, 11**] who, in 1930, first tried to transfer results in differential geometry of Riemannian manifolds to complex manifolds. In their papers there appeared a Hermitian space with the so-called symmetric unitary connection. The space with the same connection was also found independently by E. Kähler [**8**], and such a space is now called a Kaehlerian manifold. Since then, Kaehlerian manifolds have been studied extensively. Many important results have been obtained.

The study of complex submanifolds of Kaehlerian manifolds from a differential geometric point of view (that is, with emphasis on the Riemannian metric) was initiated by E. Calabi and others more than 30 years ago. Such a theory has become a very active branch of modern differential geometry in the last two decades. In particular, many important results on complex submanifolds in complex-space-forms have been obtained.

Besides complex submanifolds of Kaehlerian manifolds, there is another important class of submanifolds called totally real submanifolds. The theory of totally real submanifolds was initiated only about ten years ago. It happens that totally real submanifolds and complex submanifolds of $\mathbf{CP}^m$ are exactly those submanifolds of $\mathbf{CP}^m$ which are invariant under the curvature transformation. The theory of totally real submanifolds has undergone a very rapid development in the last decade.

In 1978, by combining the notions of complex submanifolds and totally real submanifolds, A. Bejancu [**1**] introduced the notion of CR-submanifolds as follows:

Let M be a Kaehlerian manifold with almost complex structure J. A submanifold N of M is called a CR-submanifold of M if there is a differentiable distribution $\mathfrak{D}$: $x \mapsto \mathfrak{D}_x$ on N such that

(i) $\mathfrak{D}$ is a holomorphic distribution, that is, $J\mathfrak{D}_x = \mathfrak{D}_x$ for each x in N, and

(ii) the complementary orthogonal distribution $\mathfrak{D}^\perp$ of $\mathfrak{D}$ is a totally real distribution, that is, $J\mathfrak{D}_x^\perp \subset T_x^\perp N$ for each x in N, where $T_x^\perp N$ is the normal space to N at x.

It is clear that a CR-submanifold is reduced to a complex submanifold (resp., a totally real submanifold) if $\mathfrak{D} = TN$ (resp., $\mathfrak{D}^\perp = TN$). CR-submanifolds have some fundamental properties given as follows:

In early 1978, the reviewer proved that the totally real distribution of a CR-submanifold of a Kaehlerian manifold is always completely integrable and its maximal integral submanifolds are totally real submanifolds [**2, 5, 6**]. This integrability theorem was further generalized to CR-submanifolds in a larger class of Hermitian manifolds, including the well-known Hopf manifold, by D. E. Blair and the reviewer [**4**]. For the holomorphic distribution $\mathfrak{D}$ of a CR-submanifold N in a Kaehlerian manifold, the reviewer also proved that $\mathfrak{D}$ is always a minimal distribution. Using these, it is proved that if N is closed and the de Rham cohomology group $H^{2k}(N) = 0$ for some $k \leqslant \dim_{\mathbf{C}} \mathfrak{D}$, then either the holomorphic distribution is not integrable or the totally real distribution is not a minimal distribution. For the holomorphic distribution, A. Bejancu proved that $\mathfrak{D}$ is completely integrable if and only if the second fundamental form h of N satisfies $h(X, JY) = h(JX, Y)$ for any vector fields X, Y tangent to N. Although the theory of CR-submanifolds was initiated only five years ago, many differential geometers have already contributed many results to the theory.

Kaehlerian manifolds are even dimensional. There is an odd-dimensional analogue to them, namely Sasakian manifolds, defined as follows:

Let M be an odd-dimensional Riemannian manifold with metric tensor g and let ϕ, ξ, η be a tensor field of type (1, 1), a vector field, and a 1-form on M, respectively, such that

$$\phi^2 X = -X + \eta(X)\xi, \qquad \phi\xi = 0, \qquad \eta \circ \phi = 0,$$
$$\eta(\xi) = 1, \qquad d\eta(X, Y) = g(\phi X, Y),$$
$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \qquad \eta(X) = g(X, \xi),$$

for any vector fields X, Y tangent to M. Then M is said to have a contact metric structure. If, moreover, the structure is normal (that is, the Nijenhuis torsion T formed with ϕ satisfies $T + d\eta \otimes \xi = 0$), then M is called a Sasakian manifold. Sasakian manifolds are known to admit many properties similar to those on Kaehlerian manifolds.

By using the contact metric structure on a Sasakian manifold, one may define invariant submanifolds, anti-invariant submanifolds, and contact CR-submanifolds of Sasakian manifolds corresponding to complex submanifolds, totally real submanifolds, and CR-submanifolds of Kaehlerian manifolds. There exist some natural relations between submanifolds of a Sasakian manifold and those of a Kaehlerian manifold via the Boothby-Wang fibration.

The book under review gathers and arranges mainly those results on CR-submanifolds of Kaehlerian manifolds and contact CR-submanifolds of Sasakian manifolds obtained by the authors in the last few years.

In Chapter I the authors recall fundamental ideas, definitions and formulas in the theory of Riemannian, Kaehlerian, and Sasakian manifolds. They also give some general results on the f-structure of K. Yano.

In Chapter II they state general formulas on submanifolds. The formula on the Laplacian of the second fundamental form is included. They consider submanifoldss of Riemannian space forms, especially those of spheres, and prove various theorems under conditions that the submanifolds are minimal, have parallel mean curvature vector, the normal connection is flat, or the second fundamental form is parallel.

In Chapter III contact CR-submanifolds of Sasakian manifolds are defined and studied. In particular, contact CR-submanifolds with parallel mean curvature vector or minimal contact CR-submanifolds are considered.

Chapter IV defines and studies CR-submanifolds of Kaehlerian manifolds. Integrability theorems, and also the notion of semiflat normal connection are given. Some theorems are proven under the assumption that the normal connection is semiflat, and theorems are given on CR-submanifolds with parallel mean curvature and those on minimal CR-submanifolds.

In Chapter V, applying the method of Riemannian fibre bundles, the authors investigate the relations between submanifolds of Sasakian manifolds and those of Kaehlerian manifolds.

In the last chapter, the authors consider real hypersurfaces of complex space forms. They state fundamental formulas and results on real hypersurfaces and give a theorem on pseudo-Einsteinian real hypersurfaces. They then give some theorems on minimal CR-submanifolds of $\mathbf{C}P^m$ with $\dim \mathfrak{D}^\perp = \dim T^\perp N$. Finally, they characterize certain kinds of real hypersurfaces by restrictions on the Ricci curvature or sectional curvature.

In summary, the authors did a very good job of arranging many results on CR-submanifolds and contact CR-submanifolds in one place. The book is well written. For readers who wish to do further research, the authors include an extensive bibliography containing many papers on this subject. This book should be a valuable addition to most libraries.

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Basic theory of algebraic groups and Lie algebras, by Gerhard P. Hochschild, Graduate Texts in Mathematics, vol. 75, Springer-Verlag, New York, Heidelberg, Berlin, 1981, viii + 267 pp., $32.00. ISBN 0-3879-0541-3

Linear algebraic groups, by T. A. Springer, Progress in Mathematics, vol. 9, Birkhauser, Boston; Basel, Stuttgart, 1981, x + 304 pp., $20.00. ISBN 3-7643-2039-5

A unified theory of linear algebraic groups emerged only in the 1940s. Before that time, special classes of algebraic groups such as the orthogonal groups and the general linear groups had been carefully studied, but these were often viewed separately and independently rather than as parts of some greater whole. In his *Théorie des groupes de Lie* [7], Chevalley laid the foundations for this more general theory. He developed the subject in the spirit of classical Lie theory by associating to each group its Lie algebra and by utilizing a formal exponential mapping from the Lie algebra to the group. Unfortunately, this process of linearization only worked well when the base field was of characteristic zero. At least one important result, the Lie-Kolchin theorem, did hold for