wishes to learn some quantum mechanics [3, 4]. The book serves neither purpose; rather, it seems to be for the mathematician who wishes to study Schrödinger operators for their own sake. But in this regard the book is more introductory, not nearly so penetrating as other works on the subject, for example the books of Simon [5, 6], Glimm and Jaffe [7], and especially the highly readable, comprehensive treatises of Reed and Simon [8]. The author has been parsimonious with references, particularly in the text, which could frustrate the reader who wishes to pursue the literature further.

Quantum mechanics is a little like its contemporary, Stravinsky's music—very much a part of the standard repertoire and still very revolutionary. The book comes down on the side of repertoire—functional analysis, subheading Schrödinger operators. My guess is that most readers would want a larger perspective, a glimpse of where these operators come from, and why the subject is still provocative.

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Hyperbolic boundary value problems, by Reiko Sakamoto, Cambridge Univ. Press, New York, New York, 1982, viii + 210 pp., $34.50. ISBN 0-5212-3568-5

From its beginning with the study of the vibrations of a stretched string in the eighteenth century, the theory of hyperbolic differential equations has always stood a little apart from the general theory of linear partial differential equations. This was evident in d'Alembert's famous solution formula, in which the wave forms appear explicitly as real function values in such a way as to make natural the ideas of characteristic lines, domains of dependence, and regions of influence. These motifs have carried though the times of Riemann, Goursat, and Hadamard, whose monograph on Cauchy's Problem (the initial value problem) for the wave equation led to the formulation of correctly set or well-posed initial and boundary value problems in mathematical physics. Through its connection with wave propagation and with the special and the general theories of relativity, the hyperbolic theory has played a part in many branches of science, and in particular, has helped lead to far reaching conceptions in modern cosmology. In his treatise published in 1963, Hörmander devotes separate chapters to hyperbolic equations with constant and with variable coefficients, giving a theory of hyperbolic equations and initial value problems on a level with, but distinct from, the general and the hypo-elliptic theories.

The work under review, *Hyperbolic boundary value problems* by Professor Reiko Sakamoto, is the most detailed and comprehensive study yet published of the general initial and boundary value problem for linear hyperbolic equations. The book is an English edition, translated by K. Miyahara, with revisions by the author, of an edition published in Japanese by Iwanami Shoten Publishers, Tokyo in 1978. The only comparable work known to the reviewer is the Amer. Math. Soc. Memoir **112** (1971) by Balaban, which treats the general variable coefficient mixed problem in a slightly different way.

In such a "mixed" problem with initial values given on a spacelike manifold and suitable boundary values given on a timelike (or partially timelike) manifold, the difficulties of both the hyperbolic and the elliptic problems must be confronted together. Here the hyperbolic theory can now profit from recent advances in elliptic boundary value problems, in the construction of pseudo-differential operators, and, in particular, Fourier-Laplace transforms in the complex domain. The book is typical of the flourishing modern Japanese school of work on hyperbolic problems and displays great competence, clarity at a high technical level, and the thoroughness that is now necessary in what has become a very elaborate theory.

The goal of the book is the demonstration of existence and uniqueness of solutions of general initial and boundary value problems for linear hyperbolic partial differential equations with constant or suitable variable coefficients. The problems are assumed to be $\mathfrak{S}$-well posed, in a sense involving the magnitudes of derivatives of all orders, and two necessary conditions for this involving the Lopatinski determinant of the boundary conditions are discussed in detail. The necessary compatibility conditions of initial and boundary data at the corner, or intersection, of the initial and boundary manifolds, are also assumed. This last assumption is perhaps typical of the author's approach to the subject: she concentrates totally on the analytically most sophisticated form of the problem and sets aside without comment the physically realistic case when the compatibility conditions are not satisfied. This case can be treated by means of a piecewise analytic expansion related to the sheets of the characteristic surfaces issuing from the corner locus.

The first twenty one pages of Chapter 1 are relatively elementary; indeed the reviewer considers that a reader not already acquainted with this material on the vibrating string and membrane is very unlikely to be able to follow the rest of the book.

Thereafter, the author writes as a mathematician for those prepared for the rigours of mathematics. For the achievement of the main goal, these rigours cannot be avoided, and the author from time to time provides motivating remarks which are very helpful. However, more such comments would also have been welcome.

The second chapter is devoted to the constant coefficient form of the problem for regular hyperbolic equations of arbitrary order. By means of the Fourier-Laplace transform a solution is constructed and the necessary algebraic geometry and analysis in the complex domain are carried out. Uniqueness is shown by consideration of the adjoint problem, but no reference is made to establish the basic 'Fredholm alternative' reasoning that underlies this relationship.

In the third and final chapter the general linear problem with variable coefficients is treated by means of integral inequalities or estimates. Here the relationship between the existence of solutions and the uniqueness of solutions for the adjoint problem is used in the opposite order, since uniqueness is an immediate consequence of the estimates. Thus the existence theorem in this general case has an indirect and abstract form, demanding from the reader a sure grasp of the whole machinery of proof to be fully convincing. A notable feature of the proof is the author's skilful handling of the bilinear boundary forms, or Bezout forms, required for the integral estimates and the formulation of adjoint problems. The chapter as a whole has the character of a tour de force employing varied techniques from the 'high technology' of modern mathematics now used in the study of linear partial differential equations. In a sense, therefore, the historical gap between the hyperbolic and the other linear types of partial differential equations has now been closed by the creation of this general theory of initial and boundary value problems. To have brought hyperbolic equations back 'into the fold' is an achievement that should ensure a proper place in the history and literature of the subject for the work described in this book.

Now, however, a caution is in order. As the author notes in her preface, the book gives the proofs and not the wider perspective in which the results should be reviewed. For an individual to read through the book, much preparation and preliminary study will be necessary. If the book is used in a course, much additional explanation and auxiliary work should be included. The bibliography on page 203 provides a start to the assembling of the necessary background material.

The format of the book is pleasant and the printing on the whole remarkably good. Many of the complicated displayed formulas and estimates must be read with care, and there are some places where minor crowding of the first and second order subscripts occurs. A few misprints involving rounded parentheses and pointed brackets were detected, but there was never any doubt as to the meaning intended.

To summarize, Sakamoto's book records proofs of an important recent chapter in the study of linear hyperbolic partial differential equations, and will be a valuable reference for all who wish to know the topic.

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