problem. Roughly, if operators H_0 and S are given and they commute, then there exists an operator H; however, H is highly nonunique in this general setup.

Finally, in Part IV, well past the midpoint of the book, come the first theorems which assert the existence of the limits (3). Of course, the groundwork has been carefully laid, so the proofs are now efficient. There are chapters on stationary methods, time-falloff methods, trace-class methods, and smooth perturbations. These methods are compared, and the main applications to different operators are given. In Part V the authors discuss formulas for the scattering amplitude $\hat{S}(\lambda)$. This means $\mathscr{H}$ is represented as on L^2 space (in the "frequency" variable λ) with vector values so that the free Hamiltonian H_0 is multiplication by λ. Then S is given by the operator-valued function $\hat{S}(\lambda)$, which has a meromorphic continuation into the complex plane. There is a brief discussion of its poles, the phase shift, and the scattering cross section.

The authors give us a very well-organized and complete exposition of the most abstract parts of scattering theory, to which they have been active contributors. There are historical notes and a very extensive bibliography. Two things need improvement: the index is so small as to be almost useless (although the table of contents is detailed), and the quality of the paper is poor. The book is meant to be read systematically. If you really want to understand a proof, you have to do a backwards search. Thus, the book is a very useful contribution to the literature, in that it brings together all aspects of the abstract theory, but most nonspecialists would find it too detailed. The authors wrote that "applications of the general results to special scattering systems... are treated mainly for purposes of illustration". They are presented very concisely and in small print. But the juiciest parts of the subject lie in the applications to differential operators, and so the book lacks a certain balance. The student who reads it might come away with the impression that scattering theory is a special branch of operator theory, that it is entirely concerned with the existence of the limits (3), and that its only serious application is to quantum scattering. Therefore this book should be supplemented by references which focus on other aspects of scattering theory and on its physical roots.

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Éléments de mathématique. Algèbre commutative, by N. Bourbaki, Chapitres 8 et 9, Masson, Paris, 1983, 200 pp., 150F. ISBN 2-2257-8716-6

Dimension theory of local rings, originated by Krull [**1**], can be regarded as the start of the theory of local rings. Since then many authors have contributed to the theory of local rings, including Chevalley [**2**, **3**], Cohen [**4**], Samuel [**5**], Serre [**6**] and Nagata [**7**, **8**]. Cohen proved the structure theorem of complete local rings, and Samuel constructed a good multiplicity theory of local rings.

Chapter 8 is much concerned with Samuel's multiplicity theory; Chapter 9, with Cohen's structure theorem.

As in the other issues of *Éléments de mathématique*, (1) many notions are defined under generalized situations, and (2) each chapter contains many exercises. Thus, Chapter 8 begins with the definition of the Krull dimension of a topological space. The definition obviously corresponds to the definition of the Krull dimension of a commutative ring. Namely, the Krull dimension of a topological space is defined by considering lengths of chains of irreducible closed subsets (an irreducible closed subset is a closed set which cannot be the union of two closed proper subsets) instead of lengths of chains of prime ideals in the case of commutative rings. Similarly, as a corresponding notion to the height of an ideal, the notion of codimension of a closed subset in a topological space is introduced. These are good notions in the case where the topological space X is Spec(A) with a commutative ring A (Spec(A) is the set of prime ideals of A with the Zariski topology). But, for instance, the Krull dimension of a Euclidean space of positive dimension is zero, because, in this case, a nonempty closed subset is irreducible if and only if it consists of one point. Then the Krull dimension of a commutative ring A is defined to be equal to the Krull dimension of Spec(A), and the height of an ideal I of A is defined to be equal to the codimension of the closed set defined by I in Spec(A). After some discussion on these notions in the case of finitely generated algebras over fields, the Krull dimension of a neotherian ring is discussed in §3, which is directed toward the multiplicity theory. In the remaining part of the chapter, basic parts of the multiplicity theory are presented.

Let A be a local ring with maximal ideal m. Then the multiplicity of an m-primary ideal q is introduced by Samuel [**5**] as follows: Consider the length of an A-module A/m^n for $n = 1, 2, \ldots$. Then there is a polynomial $f(x)$ with rational coefficients so that length $a/m^n = f(n)$ for large n. Then $\deg f(x) =$ Krull dim A. Take the coefficient c_0 of the ($\deg f(x)$)-term of $f(x)$. Then $(n!)c_0$ is a natural number, defined to be the multiplicity $e(q)$ of q. This notion of multiplcity has been generalized to some more general cases, including the case where A is a semilocal ring, or to the multiplicity of q with respect to a finitely generated A-module, as presented in this book. The multiplicity theory is important in algebraic geometry in view of the multiplicity of a singular point and intersections of cycles on an algebraic variety.

Chapter 9 begins with a detailed exposition of the theory of Witt vectors, including the theory of rings of Witt vectors of finite lengths. §2 begins with the definition of a p-ring; here p is a prime number, and a p-ring C is a complete local ring such that pC is its maximal ideal. Then topics related to the structure theorem of complete local rings are discussed in the unequal characteristic case (i.e., the case where the local ring in consideration does not contain any field). Namely, the structure theorem in this case asserts that if A is a complete local ring with maximal ideal m and if the characteristic p of A/m is positive, then there is a p-ring C and indeterminates $X_1, \ldots, X_m$ so that A is a homomorphic image of the formal power series ring $C[[X_1, \ldots, X_m]]$. §3 gives the structure theorem in the equal characteristic case (i.e., the case where the local ring in consideration contains some field). The structure theorem in this case asserts that if A is a complete local ring with maximal ideal m and if the characteristic of A/m coincides with that of A, then there is a subfield C of A such that C forms a complete set of representatives for A/m. Thus, A is a homomorphic image of a formal power series ring over the field C.

Cohen's structure theorem in these forms was a remarkable development in the theory of local rings, and some of the results derived from it are given in §4. Namely, §4 contains applications of the structure theorem to the theories of Japanese rings and Nagata rings; a Japanese ring is a neotherian integral domain A such that for any finite algebraic extension L of its field of fractions, the integral closure of A in L is a finite A-module. A Nagata ring is a noetherian ring A such that, for any prime ideal P of A, A/P is a Japanese ring, namely a pseudogeometric ring in the sense of Nagata [8]. At the end of the chapter there is an appendix in which a special type of extension of a local ring—roughly speaking, residue field extension—is discussed. If A is a local ring with maximal ideal M and if B is an extension discussed here, then MB is the maximal ideal of B and B is flat over A.

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Classical potential theory and its probabilistic counterpart, by J. L. Doob, Grundlehren der mathematischen Wissenschaften, vol. 262. Springer-Verlag, New York 1984, xxiii + 846 pp., $58.00. ISBN 0-3879-0881-1

It had been known for more than ten years that Doob was writing a book on this subject. Now that it has appeared, it entirely fulfills our expectations: it is a great work. Great by its dimensions, written with extreme love and care, concentrating the knowledge of a generation which was supreme in the history of potential theory, it also represents the achievement of Doob's own epoch-making research on the relations between classical potential theory and the theory of Brownian motion.