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Unified integration, by E. J. McShane, Academic Press, Orlando, Fl., 1983, xii + 607 pp., $55.00. ISBN 0-12-486260-8

This book is intended to provide a unified theory of integration (Riemann, Lebesgue, etc). Professor McShane believes that the way integration is taught now is not very efficient, since we first teach our students the Riemann integral, and then, once we have introduced them to the Lebesgue integral, abandon all our earlier work about the Riemann integral.

In this book Professor McShane tries to produce a unified way of defining the integral; of course, he includes, as a special case, the Riemann integral. The author hopes that this way of introducing the integral "can also go from beginning calculus to the graduate level without ever abandoning earlier work and starting again (as usually now happens when Lebesgue integration is met)".

The book under review begins with the introduction of the gauge integral. Some preliminaries are necessary for its definition.

An *allotted partition* of a set B in R is a finite set of pairs

$$P = \{(x_1, A_1), \dots, (x_k, A_k)\}$$

in which the A_i are pairwise disjoint left open intervals in R, the x_i are points in $[-\infty, +\infty]$, and $B = \bigcup_{i=1}^k A_i$.

To an allotted partition P and an extended real-valued function f, $S(P, f)$ will denote the sum $\sum_{i=1}^k f(x_i)m(A_i)$, where $m(A_i)$ is the length of the interval A_i.

A *gauge* Γ on a set B in $[-\infty, +\infty]$ is a function $x \to \Gamma(x)$ such that, for each x in B, $\Gamma(x)$ is a neighborhood of x.

If Γ is a gauge on $[-\infty, +\infty]$ and P is an allotted partition of a set B that is equal to the union of the A_i's, P is said to be Γ *fine* if, for each $i = 1, 2, \dots, k$, the closure of A_i is included in $\Gamma(x_i)$.

Now, the definition of the gauge integral is given as follows:

DEFINITION. Let B be a subset of R and f a real-valued function defined on a subset D of $[-\infty, +\infty]$. Suppose B is contained in D. Define g to be equal to f on B and to be zero on $[-\infty, +\infty] \setminus B$. The function f is said to be *gauge integrable* on B and of gauge integral J if for every $\varepsilon > 0$ there exists a gauge Γ on $[-\infty, +\infty]$ such that if P is a Γ fine partition of R, $S(P, g)$ exists and $|S(P, g) - J| < \varepsilon$.

This definition is used and extended stage by stage throughout the book to amplify its scope of applications. On p. 43, Theorem 7.2 shows that if f is Riemann integrable on an interval $[a, b]$, then f is gauge integrable on $[a, b]$, and its Riemann integral is equal to its gauge integral. The characteristic function of the rationals on $[0, 1]$ is gauge integrable and has zero as its gauge integral. Therefore, the gauge integral is more general than the Riemann integral; this will become more evident when it is shown (p. 296) that the gauge integral is indeed equivalent to the Lebesgue integral.

Chapter I is devoted to the introduction of the gauge integral and its relation to the Riemann integral. Chapter II is concerned with the convergence theorems, the introductions of sets of measure zero, and properties that hold almost everywhere. Absolute continuity and differentiation under the integral sign are also treated in Chapter II. Chapter III gives some applications to differential equations and to probability theory. A more general integral is defined with respect to a regular nonnegative additive measure on the left open intervals of R. Here, also, the notions of measurable functions and of measurable sets are defined. Chapter IV deals with integration in R^n. Chapter V treats line integrals and the areas of surfaces. In Chapter VI, L_p spaces, Fourier series, Fourier transforms, and special polynomials are introduced. Chapter VII gives the classical approach to measure theory.

The author begins Chapter VII by declaring

> The developments of integration theory in the preceding chapters is only one of many ways of approaching the subject. It was set forth in the belief that it is especially easy for a beginner to comprehend and is well suited for teaching a student of physics, chemistry, or engineering enough integration to be of clear benefit.

Hence, one can see that McShane's intent is to teach nonmathematics majors the powerful Lebesgue integral at once, without having to go first through the Riemann integral and he wants to do it in an elementary way so they can use it in their respective fields. That is why McShane's book is quite inclusive and covers large parts of topics found in advanced calculus, applied mathematics, probability theory, analysis, etc. The reviewer also thinks that Professor McShane wanted to prove that his definition of the integral can be applied in all the topics covered in this book.

McShane's definition of the gauge integral can be understood by someone with little background; one only needs to know the notions of neighborhood of a point, closure of a set, etc.—roughly speaking, a little topology of the real line. But a question can be asked: Does someone with that background have enough mathematical maturity to grasp all the theory developed throughout the book? The reviewer believes that most of the time he or she does not.

Another question one should look at carefully when developing a theory of integration is, how easy are the proofs of the convergence theorems? For example, the proof of the monotone convergence theorem (Theorem 4-2, p. 86) took four pages; however, the same proof (Theorem 2-10, p. 552) in Chapter VII, where the integral is defined the classical way, is more natural, shorter, and easier. In fact, McShane acknowledges that the measure was only defined on intervals and "was assumed only to be finitely additive and nonnegative, and it was that proof of the monotone convergence theorem that brought countable additivity into the theory ". The point the reviewer would like to make is that starting from an elementary definition does not always protect someone from paying the price later on in the theory.

This book is very well written, contains some very good exercises, and proofs are given in full detail. It is an honest attempt by somebody who loves measure theory to try to make this very important tool (the Lebesgue integral) accessible to a wide audience.

How well this book will succeed in achieving its avowed purpose of making the unified treatment of integration widely accepted is perhaps better judged by how fast and how often this book, or similar books, will make it to the classroom.

ELIAS SAAB

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Group representations and special functions, by Antoni Wawrzyńczyk, Mathematics and its Applications, D. Reidel Publishing Co., Dordrecht/Boston/Lancaster, 1984, xvi + 687 pp., $119.00 US; Dfl. 320,--. ISBN 90-277-1269-7

This huge book, a translation of the 1978 Polish original [**1**], is clearly intended by the author to be a study of the relations between the representation theory of groups and the special functions of mathematical physics. What has emerged is somewhat more restricted: a detailed and extensive study of the theory of spherical functions and harmonic analysis on symmetric spaces, and the application of these theories to certain special functions. The so-called special functions of mathematical physics are those useful functions which arose when physicists obtained explicit solutions of the partial differential equations governing physical phenomena—e.g., the heat, wave, Helmholtz, and Schrödinger equations—through separation of variables. With the development of quantum mechanics in the 1920s and 1930s, it became evident that there were relations between the symmetries of the partial differential equations and some of the special functions that arose as solutions of these equations. However, the first clear formulation of such a relationship appears in Eugene Wigner's 1955 unpublished Princeton lecture notes. The first extensive published treatment of the theory was the 1965 monograph of N. J. Vilenkin in which the achievements of the Gel'fand school in the theory of spherical functions were utilized [**2**]. This was followed by J. D. Talman's book in 1968, based on Wigner's lectures [**3**]. In these works the special functions occur as matrix elements of irreducible representations of the fundamental symmetry groups of physics. The matrix elements are defined with respect to a