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Nonparametric functional estimation, by B. L. S. Prakasa Rao, Probability and Mathematical Statistics, A Series of Monographs and Textbooks, Academic Press, Orlando, Florida, 1983, xiv + 522 pp., $70.00. ISBN 0-12-564020-X

The author begins his work with the observation that "Recently a large class of nonparametric methods have been developed for the estimation of distribution functions, density functions, etc., for data of several types. This book is a survey of the methods developed in the area." It is interesting to remember that the nonparametric estimation of the distribution function first carried out by Graunt [**2**] in 1662 predates parametric estimation. Moreover, contemporary interest in nonparametric density estimation goes back to a paper [**4**] written by Rosenblatt in 1956 and another written by Parzen [**3**] in 1962. However, Prakasa Rao is quite correct in noting that the explosion of output in the area has really occurred relatively recently, say in the last five to ten years.

The author covers the high points of the material in some 700 articles, dissertations and books. The fact that he has made some significant omissions simply indicates the vastness of the literature in nonparametric probability density estimation. The main point of the book is a survey and synthesis of the work in the nonparametric estimation of one-dimensional densities. In addition, the author has wisely included a number of topics in the related area of parameter estimation of stochastic processes. The inclusion of extensive problem sets at the ends of the chapters, unusual in a reference book, is an unexpected bonus.

Although the book is written at a high level of mathematical sophistication, it is presented in a clear manner which desires to communicate rather than to impress. The author makes no particular attempt to categorize some areas of investigation as fruitful, others as false trails. Inasmuch as there have been a number of rather disappointing (in retrospect) false trails blazed in nonparametric density estimation, it would appear almost obligatory for the author of a survey volume like this one to give the reader some benefits of his experience as to the strengths and weaknesses of each approach lest, at some future time, the unwary be encouraged to start down some futile path. Prakasa Rao's task is explication rather than guidance.

In those cases where several authors have written on similar topics, Prakasa Rao integrates the work, frequently improving on earlier theorems and proofs. A worker in nonparametric density estimation will find this an invaluable bibliographic source, which will save him hours trying to run down the genesis and progression of ideas. However, the author's tendency, on occasion, to put his references in a fine print collage at the end of the chapter rather than inserting them at the relevant points within the text will make it sometimes difficult to decide who did what.

The main shortcoming of the book (one which is also typical of most current work in the area) is its focus on well-defined mathematical issues, such as asymptotic properties, rather than on some representational issues yet to be resolved if nonparametric density estimation is to realize its potential in the multivariate case. To illustrate this point, let us choose between two packets of information.

A. Exact pointwise knowledge of an unknown probability density function at 100 equally spaced mesh points.

or

B. A sample of 100 points randomly selected from the underlying distribution but with no direct knowledge of the density function.

For the one-dimensional case, most of us, most of the time, will choose information packet A. But as the dimensionality increases to four, say, we note that the Cartesian information of A is likely to be of only marginal utility. The 100-point mesh is now for only three values per dimension. We are likely to miss the action if we choose A. Packet B is, however, still very useful, for the random sample gives us an intrinsic reference system which will enable us to address the primary issue in nonparametric density estimation, namely, the location of regions of relatively high probability.

Yet the point of view of Prakasa Rao's book is to transform B into a rough estimate of A. Thus as a practical matter, the author's approach is essentially bound to the very low-dimensional cases, which data analysts have been handling quite satisfactorily for some time with such crude devices as the much-maligned histogram. At this point in the development of nonparametric density estimation, pointwise estimation of the density function is not a major problem. Knowing *where* to estimate the density function (i.e., finding regions of high density) is.

For the higher-dimensional density estimation case (three and above), one can attempt to seek out high-density points (or lower-dimensional manifolds) relevant to the problem at hand and, proceeding from these "origins," attempt to examine data structure relative to them (i.e., use Lucretian (spherical) rather than Cartesian systematology) [**1**]. Or, one can attempt a variety of graphical display techniques [**5**]. Both of these approaches require extensive computer work.

In spite of the fact that it is the availability of high-speed and inexpensive computing which has made nonparametric density estimation a practical reality, the author shows a real disdain for computer graphics and number crunching in general. The lack of numerical work unfortunately dates this book and makes it of more interest to the "mathematics for its own sake" reader than to those involved with the development of new algorithms for the improved analysis of data. Nevertheless, even the most utilitarian nonparametric density estimation workers will find Professor Prakasa Rao's book a valuable reference addition to their libraries. Hopefully, the rather high price of $70 will not dissuade potential purchasers of this book from its acquisition.

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Gauss, a biographical study, by W. K. Bühler, Springer-Verlag, Berlin, Heidelberg, New York, 1981, viii + 208 pp., $19.80. ISBN 0-387-10662-6

Most contemporary mathematicians would demur if asked to name the greatest mathematician of the last fifty years. However, if the period were lengthened to a hundred years, most mathematicians would become less reticent and many would mention the name of Hilbert. If the period were lengthened to two hundred years, reticence would largely disappear and almost all mathematicians would immediately give the name of Carl Friedrich Gauss (1777–1855). In addition to his top ranking in mathematics, Gauss also ranks very high in such other scientific fields as astronomy, computing science, geodesy, physics, and statistics.

In a very real sense Gauss marks the beginning of the present epoch in mathematics. Gauss was the first person to give a proof of the Fundamental Theorem of Algebra which present-day mathematicians would find acceptable and the first person to write down the unique factorization theorem for ordinary integers in the form which we use today. Gauss aimed for a standard of mathematical precision comparable to that generally accepted today, in contrast to the occasional sleight-of-hand practiced by many of his predecessors. But, aside from this modern-seeming insistence on perfection of form, his writings have an almost contemporary flavor simply because his work has had a tremendous influence in shaping the subsequent trends of mathematical research. A symposium on the Mathematical Heritage of Carl Friedrich Gauss would be an impossibility, since it would have to cover a large part of present-day mathematics, including such widely separated fields as number theory, field theory and polynomials, linear algebra, functions of a complex variable, potential theory, special functions, calculus of variations, foundations of geometry, differential geometry, probability theory, numerical analysis, and dynamical systems.

In addition to the extraordinary level of his basic mathematical talent, several other ingredients went into the making of Gauss's outstanding mathematical career. First, he received solid financial support for his study and