Let $M$ be a closed $P^2$-irreducible 3-manifold with infinite fundamental group. It is a long-standing conjecture that the universal cover $\tilde{M}$ of $M$ must be homeomorphic to $\mathbb{R}^3$. Waldhausen [Wa] proved that this is the case when $M$ is Haken. The first result of this announcement is the following generalization of Waldhausen’s result.

**Theorem 1.** Let $M$ be a closed $P^2$-irreducible 3-manifold. If $\pi_1(M)$ contains the fundamental group of a closed surface other than $S^2$ or $P^2$ then the universal cover of $M$ is homeomorphic to $\mathbb{R}^3$.

We will say that a 3-manifold is almost compact if it can be obtained from a compact manifold $N$ by removing a closed subset of $\partial N$. Then Theorem 1 is equivalent to the assertion that the universal covering of $M$ is almost compact. A natural way to attempt to generalize Theorem 1 is to show that other coverings of $M$ are almost compact. It was conjectured by Simon [Si] that if $M$ is any compact $P^2$-irreducible 3-manifold and if $M_1$ is a covering of $M$ with finitely generated fundamental group then $M_1$ must be almost compact. Simon verified this conjecture for the case when $\pi_1(M_1)$ is the fundamental group of a boundary component of $M$. Jaco [J] generalized this to the case when $\pi_1(M_1)$ is a finitely generated peripheral subgroup of $\pi_1(M)$. More recently, Thurston [Th] showed that if $M$ admits a geometrically finite complete hyperbolic structure of infinite volume then Simon’s conjecture is true. Finally, Bonahon [B] showed that any hyperbolic 3-manifold with finitely generated fundamental group is almost compact provided that $\pi_1(M)$ is not a free product.

The second result of the announcement is the following.

**Theorem 2.** Let $M$ be a closed $P^2$-irreducible 3-manifold such that $\pi_1(M)$ contains a subgroup $A$ isomorphic to $\mathbb{Z} \times \mathbb{Z}$. Then the covering $M_A$ of $M$ with $\pi_1(M_A) = A$ is almost compact.

If $M$ is Haken then the conclusion of the theorem follows immediately from Simon’s work [Si]. For the crucial condition needed in [Si] is that if $H$ is any finitely generated subgroup of $\pi_1(M)$, then $H \cap A$ is also finitely generated, and this obviously holds, as $A$ is isomorphic to $\mathbb{Z} \times \mathbb{Z}$. If $M$ is not Haken, then the version of the Torus Theorem proved by Scott [Sc] shows that some infinite cyclic subgroup of $A$ must be normal in $\pi_1(M)$. Now the following is another long-standing conjecture.
CONJECTURE 3. If $M$ is a closed $P^2$-irreducible $3$-manifold such that $\pi_1(M)$ contains an infinite cyclic normal subgroup, then $M$ is a Seifert fiber space.

If $M$ is a Seifert fiber space, then Theorem 2 is easily proved. Thus the interest of Theorem 2 is that we do not assume that $M$ is either Haken or a Seifert fiber space. We hope that our result will be a step towards proving Conjecture 3.

Now we give a brief outline of the arguments for proving Theorem 1. Suppose that $F$ is a closed surface, not $S^2$ or $P^2$, such that $\pi_1(M)$ contains $\pi_1(F)$. Then there is a map $f: F \to M$ inducing the inclusion of fundamental groups and we choose $f$ to be a least-area map in its homotopy class. This can be done in the smooth category by choosing a Riemannian metric on $M$ and then applying the existence results of Schoen-Yau [S-Y], or it can be done in the P-L category using work of Jaco and Rubinstein [J-R]. Now in the universal cover $\tilde{M}$ of $M$, the complete pre-image $\Pi$ of $f(F)$ consists of planes as $f$ injects $\pi_1(F)$ into the fundamental group of $M$, and these planes are embedded by results of Freedman, Hass, and Scott [F-H-S]. Further, the intersection of two planes never contains a circle. These planes define a division of $M$ into 3-dimensional chambers, each of which covers one of the compact chambers in $M$ obtained by cutting $M$ along $f(F)$. We show that each of the chambers in $\tilde{M}$ is simply connected and that each chamber in $M$ is $\pi_1$-injective. We also show that if $M$ is not Haken then each chamber in $M$ is a handlebody. It follows that the closure of each chamber in $\tilde{M}$ is almost compact. We show that if a finite collection $\Sigma$ of the planes in $\Pi$ are removed, then the new bigger chambers formed by cutting $\tilde{M}$ along $\Pi - \Sigma$ have the same property. As a compact set in $M$ can meet only finitely many planes of $\Pi$, it follows immediately that any compact subset of $\tilde{M}$ lies in the interior of a 3-ball. This shows that $M$ is homeomorphic to $R^3$ as required.

The proof of Theorem 2 uses similar ideas. Let $T$ denote the 2-torus and let $f: T \to M$ be a map of least area such that $f_* (\pi_1(T)) = A$. The preimage in $M_A$ of $f(T)$ is a collection of possibly singular tori and annuli. If they were all embedded in $M_A$ then we would argue much as in the proof of Theorem 1. This need not be the case, even when $M$ is a Seifert fiber space, but we show that $M_A$ must have a finite cover in which the preimage of $f(T)$ consists of embedded tori and annuli. This result completes the proof of Theorem 2.

REFERENCES


DEPARTMENT OF MATHEMATICS, HEBREW UNIVERSITY OF JERUSALEM, JERUSALEM, ISRAEL

DEPARTMENT OF MATHEMATICS, MELBOURNE UNIVERSITY, PARKVILLE, VICTORIA 3052, AUSTRALIA

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF LIVERPOOL, LIVERPOOL, ENGLAND