BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 16, Number 2, April 1987

0273-0979/87 $1.00 + $.25 per page

Strong approximation by Fourier series, by László Leindler, Akadémiai Kiadó, Budapest, 1985, 210 pp., $23.00. ISBN 963-05-4044-4

In 1900 Fejér proved his celebrated result that the first arithmetic means

$$\sigma_n(f; x_0) := \frac{1}{n+1} \sum_{k=0}^{n} S_k(f; x_0),$$

$$S_k(f; x_0) := \sum_{j=-k}^{k} \frac{1}{2\pi} \int_{-\pi}^{\pi} f(u) e^{ij(x_0-u)}\, du,$$

of the Fourier series of a 2π-periodic, (bounded) integrable function f converge to $f(x_0)$ at each point x_0 of continuity of f. This result was strengthened by Hardy-Littlewood in 1913 to the effect that even the strong means

$$h_n(f; x_0) := \frac{1}{n+1} \sum_{k=0}^{n} |S_k(f; x_0) - f(x_0)|$$

converge to zero. This initiated the field of strong summability. These assertions on pure convergence then gave rise to questions concerning rates of convergence. In 1912 Bernstein showed that $|\sigma_n(f; x) - f(x)| = \mathcal{O}(n^{-\alpha})$ uniformly for all x, provided f belongs to a Lipschitz class with $0 < \alpha < 1$. And again, the same estimate holds true even for the strong error $h_n(f; x)$ as established by Alexits-Králik in 1963. In fact, the latter result is considered to be the starting point of the very active field of strong approximation. This includes extensions to more intricate means (Cesàro-(C, γ), Riesz), saturation phenomena, conjugate functions, inverse results, embedding theorems, general orthogonal expansions, etc. On the other hand, particular emphasis is to be laid on negative results in case the assertions on strong approximation differ from the classical ones.

The present book aims to give a uniform treatment of the most important results on the strong approximation of continuous functions by (one-dimensional, trigonometrical) Fourier series. It is written by *the* expert in the field who not only contributed a number of papers, but gave many stimulating lectures on the occasion of the various conferences on approximation theory held during the last two decades. Many participants, among them the reviewer, felt the need for a monograph on the subject. In view of the additional nonlinearities arising, strong approximation is a nice piece of hard analysis. Accordingly, emphasis is laid upon a careful development of the methods of proof.

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