Carter's book contains a very good introduction to the present state of affairs and can be warmly recommended to anyone who is interested in penetrating into the highly interesting domain of finite groups of Lie type. The book has an extensive bibliography.

T. A. SPRINGER

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 17, Number 1, July 1987

0273-0979/87 $1.00 + $.25 per page

The arithmetic of elliptic curves, by Joseph H. Silverman, Graduate Texts in Mathematics, vol. 106, Springer-Verlag, New York, Berlin, Heidelberg and Tokyo, 1986, xii + 400 pp., $48.00. ISBN 0-387-96203-4

The arithmetic (= diophantine theory) of curves of genus 0 is now very well understood. That of curves of genus > 1 is still in a rudimentary and unsatisfactory state. For curves of genus 1 there is a large body of established theory and an even larger body of interrelated conjecture: the whole being currently in a state of exciting development.

We work over a ground field k, which may be the rationals $\mathbf{Q}$, or e.g., a global or local field. An elliptic curve defined over k consists of a curve of genus 1 together with a point 0 (say) on it, both defined over k (we shall often say "rational" instead of "defined over k"). Here we encounter our first puzzle. There is no known algorithm for deciding (e.g., when $k = \mathbf{Q}$) whether there is a rational point on a given curve of genus 1 or not: in particular there is no Hasse principle (local-global principle). However, to every curve of genus 1 there is associated in a canonical way an elliptic curve over the same ground field (its jacobian, a generalization of the notion from algebraic geometry). The theory of curves of genus 1 thus largely reduces to that of elliptic curves.

The points of an elliptic curve have a natural structure as an abelian group, the given point 0 being the neutral element ("zero") of the group. In fact the elliptic curves over a field k are precisely the abelian varieties of dimension 1 over k. In particular the set of rational points has a natural abelian group structure. When $k = \mathbf{Q}$ a famous theorem of Mordell states that this group is finitely generated. This result was generalized by Weil and others and the group is usually called the Mordell-Weil group (for the given elliptic curve and ground field). There is, however, as yet no algorithm for determining the Mordell-Weil group, though this can usually be done in specified cases. The absence of an algorithm here is closely associated with the failure of the Hasse principle mentioned above. The "obstruction" to the Hasse principle is encapsulated in a group discovered independently by Tate and Shafarevich and called the Tate-Shafarevich group. It has many interesting properties, both proved and conjectural. Without doubt the reviewer's most lasting contribution to the theory is the introduction of the cyrillic letter Ш ("sha") to denote this group, a usage which has become universal.

When $k = \mathbf{Q}$ early workers appear to have believed that the rank (number of generators of infinite order) of the Mordell-Weil group is bounded; now the opposite is conjectured, but the truth is not known. The possible structures of the torsion part of the Mordell-Weil group have, however, been determined by Mazur using tools from the theory of modular forms.

Associated with an elliptic curve over a global field such as $\mathbf{Q}$ there is associated an L-function, many of whose properties remain conjectural. Guided first by a heuristic intuition and then by massive numerical computations, Birch and Swinnerton-Dyer were led to some very precise conjectures relating the behavior of the L-function to such things as the Mordell-Weil and Tate-Shafarevich groups. These conjectures have stood the test of much subsequent investigation, but it is only recently that some fragments of them have been proved.

The above partial account indicates the central position of the theory of elliptic curves and the wide variety of disciplines on which it draws. The author justifiably remarks "Considering the vast amount of research currently being done in this area, the paucity of introductory texts is somewhat surprising." In the reviewer's opinion his book fills the gap admirably. An old hand is hardly the best judge of a book of this nature, but the reports of graduate students are equally favorable.

J. W. S. CASSELS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 17, Number 1, July 1987

0273-0979/87 $1.00 + $.25 per page

Geometry of CR-submanifolds, by Aurel Bejancu. Mathematics and Its Applications, D. Reidel Publishing Company, Dordrecht, Boston, Lancaster, and Tokyo, xii + 169 pp., $49.50. ISBN 90-277-2194-7.

The study of complex submanifolds of a Kaehlerian manifold, in particular, of a complex projective space, is one of the most important fields in differential geometry. It began as a separate area of study in the last century with the investigation of algebraic curves and algebraic surfaces in classical algebraic geometry. Included among the principal investigators are Riemann, Picard, Enriques, Castelnuovo, Severi, and C. Segre. It was J. A. Schouten, D. van Dantzig and E. Kähler [**5**, **8**, **9**] who first tried to study complex manifolds from the viewpoint of Riemannian geometry in the early 1930s. In their studies, a Hermitian space with the so-called symmetric unitary connection was introduced. A Hermitian space with such a connection is now known as a Kaehlerian manifold.

It was A. Weil [**10**] who in 1947 pointed out that there exists in a complex manifold a tensor field J of type $(1,1)$ whose square is equal to minus the identity transformation of the tangent bundle, that is, $J^2 = -I$. In the same year, C. Ehresmann introduced the notion of an almost complex manifold as