they are not); examples, originally due to Grima, of distinct fibered links with the same Alexander polynomial, and examples of nontrivial links with trivial fibering of the complement. Most of these are easily computed using the splice diagram.

One general pattern in the study of topological properties of algebraic varieties is that some property is first discovered using the algebraic structure, and later reproved in a geometric or topological setting. Eisenbud and Neumann do this as well: They show geometrically that the complement of algebraic link fibers, the eigenvalues of the monodromy on homology are roots of unity, and that its largest Jordan block associated to the eigenvalue one is of size one. (The first two facts already had geometric proofs; the last one had been proved analytically by Steenbrink and later Navarro, using mixed Hodge theory. They also obtained a related result in higher dimensions.)

My only major complaint with the book is that there is no index. An index should be easy to make in our present computer age (and was easy even in the past, when a student could be hired to do it). The lack of an index reduces the value of a book as a reference. I suspect that an index would help to organize such matters as definitions as well. For example, the important term 'solvable link' appears to be defined in the statement of Theorem 9.2, and the central term 'graph link' appears not to be formally defined at all; it is first mentioned in the body of the text in §8, without definition, but fortunately had already been defined—in passing—in the introduction. The many favorable qualities of this book more than make up for these defects, though. It is an excellent book for students to read, especially because of its collection of scattered and occasionally unknown results, for example the algorithm for passing from the Puiseux expansion to the polynomial. And lastly, there is a lengthy and superb introduction.

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Methods of representation theory with applications to finite groups and orders, vol. II, by Charles W. Curtis and Irving Reiner. John Wiley and Sons, New York, Chicester, Brisbane, Toronto, Singapore, 1987, xv+951 pp., $95.00. ISBN 0-471-88871-0

> "... while, in the present state of our knowledge, many results in the pure theory are arrived at most readily by dealing with the properties of substitution groups, it would be difficult to find a result that could be most directly obtained by the consideration of groups of linear transformations."

These words were penned by William Burnside, F.R.S., Professor of the Royal Naval College, Greenwich, as part of the preface to the first edition of *Theory of groups of finite order* [**B1**] in June of 1897. He was explaining why his new book included no representation theory. A representation of a finite group G is a homomorphism $\rho: G \to \mathrm{GL}(V)$ where $\mathrm{GL}(V)$ is the group of all nonsingular linear transformations on a finite-dimensional vector space V over a field. At the time of Burnside's writing, the study of group representation (by linear transformations or linear substitutions as Burnside would have said) was in its infancy. It had been invented, at least partially, by Ferdinand Georg Frobenius in a series of papers on group characters (see [**F**, papers 53, 54, 56]). A character is the trace of a representation, but Frobenius's approach was somewhat indirect in that he did not originally define characters using representations. Rather he was generalizing the concept used in number theory of a character of an abelian group. By 1897, Frobenius's work was mostly developmental with several interesting examples but scarcely an application. Burnside was openly skeptical.

Skeptical does not mean uninterested. Certainly something in Frobenius's work fired Burnside's imagination. A few years later Burnside [**B2**, **B3**] presented a direct development of group characters from representations. His presentation is not far from what we now teach in our courses. However, more dramatic events converted Burnside totally. In the preface to the second edition [**B4**], written in March of 1911, he stated the reason for his omitting an account of group representations in the first edition "no longer holds good." More emphatically he wrote,

> "In fact it now seems more true to say that for further advances in the abstract theory one must look largely to the representation of a group as a group of linear substitutions."

In the fourteen years between the editions, Burnside's image of group theory had made an abrupt shift. We can only guess what thoughts had passed through his mind, but surely a major reason for his change of view was directly connected to one of the hot topics in finite group theory at the turn of the century. The question concerned the orders of the finite simple groups, or more generally, what conditions on the order of a group together with local information will insure that a group is solvable. Burnside called it "the most general problem in pure group theory" [**B1**, p. 343]. It is the problem of analyzing all distinct types of groups whose order is a given integer. It was the subject of the last chapter of Burnside's first edition.

As background, we may recall that Sylow's theorems imply the solvability of groups of order p^α, p a prime. In 1893 [**F**, paper 43] Frobenius proved that if the order of G is not divisible by the square of any prime, then G is solvable. Later he was able to generalize this result [**F**, paper 49]. He showed, as his main lemma, that if an integer m has distinct prime factors and if every prime factor of n is greater than the largest of those of m, then any group of order mn has exactly n elements whose orders divide n. In 1901, he applied his theory of complex characters to get that the n elements whose orders divide n form a subgroup of the group. Burnside had proved that if all Sylow subgroups of a group are cyclic then the group is solvable. Several other mathematicians also made contributions to this area.

A particularly interesting question of that time concerned groups whose orders are divisible by only two primes, p and q. Burnside proved in his first edition that any such group is solvable provided that its Sylow subgroups are abelian. He also presented the following theorem which generalized an early result, independently verified by himself and Frobenius.

> A group whose order is $p^\alpha q^\beta$, where α is less than $2m$, m being the index to which p belongs $\pmod q$, is solvable.

In 1902 Frobenius [**F**, paper 65] published a generalization of this result to include the case $\alpha = 2m$ and also the case in which G has at most p^m distinct Sylow q-subgroups. Curiously his paper ends with a one sentence paragraph.

> "Damit ist der Beweis durch rein gruppentheoretische Betrachtungen geführt, ohne jede Hülfe der Substitutionstheorie, d.h. ohne Benutzung irgend einer Darstellung der Gruppe G."

These were prophetic words, emphasizing that his result required no representation theory. Possibly in his imagination Frobenius had had a glimpse of what was to come. Two years later the following theorem was published.

> Every group of order $p^\alpha q^\beta$ is solvable [**B5**].

It has become known as Burnside's $p^\alpha q^\beta$ Theorem, and its proof was close to being pure representation theory, i.e., the character theory as developed by Frobenius. It was probably the first major theorem proved using representations of groups. Many more would follow.

It must have been frustrating for the group theorists of that day to have such a simple statement whose truth could not be demonstrated by group-theoretic techniques. In fact, Burnside's proof was essentially the only one available for approximately 60 years. In the 1960s John Thompson finally outlined a purely group-theoretic proof. David Goldschmidt has given a fairly short proof in the special case that p and q are odd primes [**G**]. But even Goldschmidt's paper is not light reading. Probably Burnside's proof is still the best for an understanding of why the theorem is true.

Thompson's proof had been developed from his work on the same problem that had concerned Burnside and Frobenius, namely the classification of the finite simple groups. He and Walter Feit contributed a colossal step in the solution when they showed that all groups of odd order are solvable [**FT**]. Representation theory played a vital rôle in the proof as well as in other parts of the classification endeavour. In the 1960s most group representation theory was still closely tied to group theory. There had certainly been many advances since Frobenius's first work. Many new ideas had been introduced. For example, Richard Brauer began his study of modular representation theory and block theory in the late 1930s. However Brauer, like many others, concentrated on character-theoretic developments and mostly regarded representations as tools for the investigation of group structures.

By the late 1970s, when the classification problem was winding toward its ultimate solution, it had become clear that representation theory had a life of its own. No longer the child of group theory, it had become suffused with ideas and techniques from many areas of mathematics. It has applications in many areas both within and outside of mathematics. At the A.M.S. Summer Institute in 1986 the participants were treated to a three-week demonstration of the diversity of the modern subject. Lectures were given on topics ranging from the classical complex characters of solvable groups to algebraic singularities, line bundles, and flag manifolds.

The most unusual feature of the books by Curtis and Reiner is that they give the reader a picture of the vast array of topics that can be labeled under the heading of representation theory. The second volume contains long chapters on K-theory and on class groups for group rings and orders, as well as somewhat shorter expositions of block theory and of the representations of finite groups of Lie type. Briefer, though still lengthy, chapters discuss rationality questions such as Schur multipliers, contributions from the representation theory of finite-dimensional algebras such as quivers and Auslander-Reiten sequences, and results on Burnside rings and representation rings of finite groups. No other text comes close to covering so much ground. However, even with two volumes of almost 1800 pages the books are nowhere near complete. Some of the most exciting areas of current research, such as cohomological methods and the connections with the representation theory of algebraic groups, are given only introductions or barely mentioned. The field has become too big.

When Curtis's and Reiner's first book [**CR**] was published in 1962 it immediately became everybody's standard reference. It was by far the best and most complete of the few texts that were available. By contrast, there are many books on representation theory in print today, but they are all somewhat specialized, highlighting only one or only a few aspects of the modern theory. At the same time most of us who work in group representations are also specialized, and for us the new volumes by Curtis and Reiner will provide access to many of those topics that are outside of our areas. The text is well written and organized, and the approach is generally up to date. The books should be easily readable by students with any sort of reasonable background in group theory and abstract algebra. As introductions and references to the numerous topics that make up modern representation theory, the volumes by Curtis and Reiner are to be highly recommended.

It is unfortunate that Irving Reiner did not live to see the publication of the second volume. He died in October of 1986 after a long illness. Reiner was a helpful and personal friend to many of us and a good friend of the mathematical community in general. We will miss him.

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Univalent functions and Teichmüller spaces, by Olli Lehto. Graduate Texts in Mathematics, vol. 109, Springer-Verlag, New York, Berlin, Heidelberg, London, Paris, New York, 1987, xii+257 pp., $46.00. ISBN 0-387-96310-3

Quasiconformal mappings have played a prominent role in geometric function theory for nearly fifty years. We recall that a sense-preserving diffeomorphism f of one plane region D onto another is called a K-quasiconformal map if its differential $f'(x)$, viewed as a linear map of $\mathbf{R}^2$ onto itself, satisfies the inequality

$$\max\{\|f'(x)u\|; \|u\| = 1\} \leq K \min\{\|f'(x)u\|; \|u\| = 1\}$$

at every point in D. A sense-preserving homeomorphism of D into the plane is K-quasiconformal if there is a sequence of K-quasiconformal diffeomorphisms that converges to f uniformly on compact subsets of D. A sense-preserving homeomorphism of one Riemann surface onto another is K-quasiconformal if all its compositions with local charts are K-quasiconformal maps in the plane. Finally, a quasiconformal map is a sense-preserving homeomorphism that is K-quasiconformal for some number $K \geq 1$. It is important to observe that 1-quasiconformal maps are conformal.

In 1939, a fundamental paper of Teichmüller introduced quasiconformal maps to the study of spaces of Riemann surfaces. Choose a compact Riemann surface X of genus $p \geq 2$ and define a pseudometric on the set of all sense-preserving homeomorphisms of X onto Riemann surfaces of the same genus by putting

$$d(f, g) = \log K \tag{1}$$

if K is the smallest number such that there is a K-quasiconformal map in the homotopy class of $g \circ f^{-1}$. The metric space that results from identifying f with g when $d(f, g) = 0$ is Teichmüller's space T_p of marked Riemann surfaces of genus p. Teichmüller proved that T_p is homeomorphic to $\mathbf{R}^{6p-6}$.