


Jean-Pierre Rosay
University of Wisconsin-Madison


As in many areas of mathematics, most of the research in noncommutative ring theory—including some basic results—is less than thirty years old. As recently as 1956, Jacobson was able to include most of the subject in his classic *Structure of rings* [J], a book of moderate size. But by the time he revised *Structure of rings* in 1964, Goldie's Theorem and other new developments had appeared, and he merely sketched them in appendices.

It is not surprising that after *Structure of rings* no one has tried to write a book covering all of ring theory. Rowen has not gone quite so far in the two volumes under review, but his *Ring theory* is the most ambitious and encyclopedic book yet written in the area. It is both a textbook and an up-to-date encyclopedia of research. Roughly speaking, the greater part of volume I covers basic structure theory, and is meant to be the basis of a graduate level ring theory course; volume II and parts of volume I treat specialized areas of ring theory in which research is active today. These include noncommutative Noetherian rings, homological algebra, polynomial identities, central simple algebras, and enveloping algebras. The one major topic omitted is the representation theory of Artin algebras.

It is useful to consider separately three potential audiences for *Ring theory*, all beyond the undergraduate level: Graduate students, for whom it will be a textbook; experts (= ring theorists), for whom it will be a compendium of recent research; and nonexperts (= established mathematicians who are not ring theorists), for whom it will be a guide to the subject. For all of them, it also serves as a reference.

As a textbook for graduate students, *Ring theory* joins the best. The only other books which I think make excellent textbooks are *Noncommutative*
rings, by Herstein [H], and Associative algebras, by Pierce [Pi]. Noncommutative rings is quite different from the other two books. It has a relatively informal style and aims more to illustrate several important ideas than to develop the subject systematically.

Associative algebras and Ring theory have much more in common with each other than with Noncommutative rings. Among basic results, both include the following: The Jordan-Hölder Theorem, the Wedderburn-Artin Theorem, radical theory, the Chevalley-Jacobson density theorem, the Krull-Schmidt Theorem, the Morita Theorems. Associative algebras is also much more than a textbook, although it is not as ambitious as Ring theory. It treats several advanced topics: Representations of Artin algebras (barely mentioned in Ring theory); division algebras (there is considerable overlap with Ring theory, but the approach in Associative algebras is more valuation theoretic and number theoretic); polynomial identities (only enough to present Amitsur’s division algebras which are not crossed products). It does not include Ore’s theorem on noncommutative localization, Goldie’s Theorem, or the theory of noncommutative Noetherian rings, which receive extensive treatment in Ring theory. As textbooks, a choice between the two is mostly a matter of taste. I prefer Ring theory because I think noncommutative localization and Goldie’s Theorem belong in a basic graduate course.

The experts will find several pleasant and attractive features in Ring theory. The most noteworthy is the inclusion, usually in supplements and appendices, of many useful constructions which are hard to locate outside of the original sources. Two examples are Brauer factor sets and the Magnus embedding of a free group in a power series ring. Another desirable feature is the excellent indexing and bibliography.

But ring theorists will find that the in-depth coverage of special topics cannot match the comprehensive treatment in monographs, devoted to a single topic. Examples are The algebraic structure of group rings, by Passman [P], Polynomial identities in ring theory, by Rowan himself [R], and Noncommutative Noetherian rings, by McConnell and Robson [M-R]. Ring theory is a useful supplement to these books in so far as it is more up to date, but for the most part researchers in ring theory are likely to prefer the specialized literature.

The audience of nonexperts, mathematicians whose speciality is not ring theory, will find Ring theory ideally suited to their needs. It treats a lot of ring theory with considerable depth in volume II, with volume I as a foundation. They, as well as students, will be well served by the many examples of rings and the glossary of major results.

Rowen has attempted a great deal in Ring theory. I believe he has been mostly successful. As a textbook, it joins a short list of the best, and for mathematicians who are not ring theorists it is the book of choice. For ring theorists, it has many strong points, but cannot replace the more specialized monographs.


In the preface of their book, García-Cuerva and Rubio de Francia say: “After the classical monograph of A. Zygmund [31], the standard references for the important developments that occurred in Fourier Analysis during the second half of this century are E. M. Stein [25] and Stein and Weiss [29], both published around 1970.” Since that date there have been dramatic advances in several areas of Harmonic Analysis. We shall describe some of these. We begin with the Theory of Hardy spaces and shall present a more elaborate description of this development since we can use some of the material presented to help explain the progress made in other areas.

Classical Harmonic Analysis in one dimension is either associated with the Real line $\mathbb{R}$ or the Torus $\mathbb{T} = [0, 2\pi)$, often identified with the Circle Group $\{z \in \mathbb{C} : z = e^{i\theta}, 0 \leq \theta < 2\pi\}$. Let us concentrate on $\mathbb{R}$, which we consider embedded in $\mathbb{R}^2$ (or $\mathbb{C}$) as the boundary of the Upper Half Plane $\mathbb{R}^2_+ \equiv \{z = (x, y) \in \mathbb{R}^2 : y > 0\} = \{x + iy \in \mathbb{C} : y > 0\}$. If $0 < p \leq \infty$ the Hardy Space $H^p$ consists of all holomorphic functions $F(x + iy)$ on $\mathbb{R}^2_+$ such that

$$
\|F\|_{H^p} \equiv \sup_{y>0} \left\{ \int_{-\infty}^{\infty} |F(x + iy)|^p dx \right\}^{1/p} < \infty.
$$

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use