In sum, the book may be highly recommended (with the caveat above) to beginners who wish a bird's-eye view of this broad and beautiful, but sometimes deep and sophisticated theory.

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Unit groups of classical rings, by Gregory Karpilovsky, Clarendon Press, Oxford, 370 pp., $98.00. ISBN 0-19-853557-0

Call a ring unitary if it has an identity element under multiplication. If R is a unitary ring, then there are several groups and monoids that are naturally associated with R. Among these are the additive group $(R, +)$ of R (that is, the group on the set R with operation the operation of addition defined on the ring R), the multiplicative monoid $(R, \cdot)$ of R, and the multiplicative group $U(R)$ of units of R. (A unit of R is an element that has a multiplicative inverse in R; for example 1 and -1 are the units of the ring of integers.) Ring theorists have long been interested in the interplay and relations that exist between the algebraic structures R, $(R, +)$, $(R, \cdot)$ and $U(R)$. Clearly R nominally determines the other three structures. What about the converse? To what extent do one or more of the structures $(R, +)$, $(R, \cdot)$ and $U(R)$ determine R? A different kind of question concerns realization: for example, given an abelian group G and a group H, can G and H be realized as the additive and unit groups, respectively, of a unitary ring R, and if so, how many realizations are there, to within isomorphism? To illustrate this last question, suppose $G = Z$, the infinite cyclic group. If G is the additive group of a unitary ring R, and if g is a generator for G, then the multiplication on R is completely determined by the integer k, where $g^2 = kg$; moreover, $k = \pm 1$ since R is unitary. Since $(-g)^2 = (-k)(-g)$, where $-g$ is also a generator for G, it follows that R is isomorphic to the ring of integers, so H must be cyclic of order two in order for the pair (G, H) to be realizable. In a similar vein, Chapter 6 of the book under review determines the unitary rings R for which $U(R)$ is cyclic. Natural variants on these themes arise if one restricts to rings or groups that satisfy a given condition E. For example, early work by Fuchs, Szele and others (see Chapter XVII of [1]) focused on the problem of determining what abelian groups could be realized as the additive group of an Artinian ring.

The book under review addresses the group $U(R)$ in several contexts where this group has traditionally been of interest. The first context in which $U(R)$ was extensively studied was that in which R is the ring of algebraic integers of a finite algebraic number field K. The principal result in this area is the Dirichlet-Chevalley-Hasse Unit Theorem, which states that $U(R)$ is the direct product of a specified group and a free abelian group of specified finite rank; in particular, $U(R)$ is finitely generated. An important special case that lies within this setting is that in which $K = Q(\varepsilon_n)$ is the cyclotomic field of nth roots of unity over the rational field Q, and hence $R = Z[\varepsilon_n]$; the author treats this case in Chapter 3. Other topics covered are the unit groups of fields, division rings and group rings; moreover, Chapter 7 is devoted to consideration of finite generation of $U(R)$.

Karpilovsky's book brings together a broad range of topics from group theory, commutative and noncommutative ring theory, field theory, and algebraic number theory. The quality of exposition in the text is quite good; the author has done a praiseworthy job in making the material accessible to a knowledgable, but nonspecialist, reader. In the process, some generality and depth of coverage has been sacrificed in order to broaden the audience for the book. Overall, the book can be highly recommended to a reader interested in learning about a wide range of topics in which unit groups have historically played a significant role.

Reference

1. L. Fuchs, *Infinite abelian groups*, vol. II, Academic Press, New York, 1973.

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Theory of reproducing kernels and its applications, By Saburou Saitoh, Longman Scientific and Technical, 157 pp., 1988, $57.95. ISBN 0-582-03564-3

Let S be a set and $\mathscr{H}$ a Hilbert space; a mapping κ from S to $\mathscr{H}: x \to k_x$ gives rise to a kernel function

$$K(x, y) = (k_y, k_x)$$