analytic, as exemplified by the different treatment of the discrete series. Finally, Knapp's book contains a large supply of examples, valuable to both novice and expert. Because of these differences, any serious reader of *Real reductive groups.*I should have Knapp's book [3], together with Vogan's book [5], close at hand.

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Pi and the AGM: *A study in analytic number theory and computational complexity,* by Jonathan M. Borwein and Peter B. Borwein. Canadian Mathematical Society Series of Monographs and Advanced Texts, John Wiley and Sons, New York, Chichester, Brisbane, Toronto, Singapore, 1987, xi + 414pp. $49.95. ISBN 0-471-83138-7

On August 15, 1989, the op-ed page of the New York Times carried an article entitled *Call it Pi in the sky* by Stewart Wills. The leading paragraph gives Wills' immediate response to the recent work of the Chudnovskys concerning the digits of π! "I shouldn't have let the news upset me. It was cause for celebration. Two Columbia University mathematicians using a powerful computer had calculated the symbol pi to 480 million decimal places. Yet it has troubled me ever since." His disturbance centers on the fact that in geometry class many years ago he thought he had nailed π down to 3.14159. However later in the article he provides comments that many professional mathematicians must have felt:

"Like any great discovery, the Chudnovskys' calculation of pi raises more questions than it answers. The least imaginative: Of what use is this behemoth?

"One article suggested that a principal benefit of the calculation would be to give the world's most powerful computers a thorough workout. The formula, then, is a sort of Nautilus machine for the Cray-2.

"But what could be lovelier, after all, than the thought of men and women pursuing a goal that recedes ever further the nearer one draws to it—the last digit of pi? The Chudnovskys' achievement was a work not of practical science but of poetry. They have coaxed from the darkness 480 million slivers of light."

Of course this is not the only possible response to such an achievement, P. Beckmann [1] wrote in April 1987 on the work of Y. Kanada et al. calculating the first 140 million digits of π:

"It takes a lot of brains to program a computer to calculate (and verify!) 140 million decimal digits in a reasonable time, and the programers have my respect and admiration—but they have it as sportsmen, not as scientists.

"The need for brains, or even the development of new techniques, is not what defines science. It also takes brains and the development of new techniques to stuff 21 people into a phone booth. And indeed, the calculation of the first 140 million digits is a feat worthy of ranking along with the other stunts in the Guiness Book of World Records.

"But I believe that when one applies the test of whether it can lead to a generalization of knowledge, this feat will be found scientifically worthless. It does nothing, for example, for determining the statistical distribution of the digits (which is very probably uniform—same frequency for any digit), for the 140 million still leave somewhat of a gap all the way to infinity; on the other hand, a mathematical derivation of that distribution, which will no doubt be eventually achieved, can use nothing from the first 140 million digits or any other special case."

Surely if Beckmann's assertions are correct, we should be disturbed by the frivolous nature of such work and less pleased by Wills' sparkling sentimentality.

Fortunately we have the Borweins' beautiful book to refute Beckmann's argument. There is both serious and beautiful mathematics underlying these massive calculations of the digits of π. The history of the theory behind the calculations is presented in Chapter 11 of the book under review. It is perhaps most succinctly summarized by the Borweins in [2, p. 112].

"Pi, the ratio of any circle's circumference to its diameter, was computed in 1987 to an unprecedented level of accuracy: more than 100 million decimal places. Last year also marked the centenary of the birth of Srinivasa Ramanujan, an enigmatic Indian mathematical genius who spent much of his short life in isolation and poor health. The two events are in fact closely linked, because the basic approach underlying the most recent computations of pi was anticipated by Ramanujan, although its implementation had to await the formulation of efficient algorithms (by various workers including us), modern supercomputers and new ways to multiply numbers."

The Borweins' book explores in the first five chapters the glorious world, so dear to Ramanujan, of classical theta functions and modular equations with an eye on producing algorithms which rapidly converge to π.

Indeed they provide the link between numerical algorithms and the arithmetic-geometric mean (AGM) early on. They begin with Gauss' iteration

$$a_{n+1} = \frac{a_n + b_n}{2} \qquad b_{n+1} = \sqrt{a_n b_n}.$$

The sequences a_n and b_n can be shown to converge extremely rapidly to a common limit (assuming $0 < b_0 \leq a_0$) denoted $M(a_0, b_0)$ which depends only on a_0 and b_0. From here Gauss identified $M(a_0, b_0)$ with a certain elliptic integral. By §2.5 the Borweins have established the link between such algorithms and rapid calculations of the digits of π. Complexity questions occupy Chapters 6 and 7. The remaining chapters provide applications and other approaches.

It seems to me that this would be a marvellous text book for a graduate course in what has sometimes been called constructive mathematics, that never-never land between mathematics and computer science. This book contains important and currently neglected topics in classical analysis together with enough material on algorithms to make it a really exciting new course. The authors surely had something like this in mind for they provide ample exercises. I strongly hope that a number of people will be inspired to use the Boreweins' book as the text for such a course.

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Cordes' two-parameter spectral representation theory, by D. F. McGhee and R. H. Picard. Pitman Research Notes in Mathematics Series, Volume 177, Longman Scientific and Technical, Harlow, United Kingdom and New York, 1988, 114 pp., \$41.95. ISBN 0-470-21084-2

Mathematical developments can be viewed as a river fed from numerous tributaries and giving rise to branching streams of vigorous activity, quiet meandering backwaters which may become brackish and stagnate or possibly return with renewed vigour to the main stream. Multiparameter spectral theory, of which McGhee and Picard's book deals with a particular but central aspect, is an example of such an analogy.

In order to discuss the central questions of multiparameter theory and its relation to other branches of classical and functional analysis it is necessary to formulate the general problem.

Suppose one has k separable Hilbert spaces H_r, $1 \le r \le k$ and a collection of linear operators T_r, V_{rs}, $1 \le s \le k$, defined on these spaces. One now forms the k linear combinations

$$W_r(\lambda) \equiv T_r + \sum_{s=1}^{k} \lambda_s V_{rs}, \qquad 1 \le r \le k \tag{1}$$

where $\lambda_s \in \mathbf{C}$ are scalars. The central question is then to determine the scalars $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_k) \in \mathbf{C}^k$ such that all the linear operators $W_r(\boldsymbol{\lambda})$ have nonzero kernels. Briefly then, we have a multiparameter spectral problem invoking a plethora of questions thus generalising in a nontrivial manner one-parameter spectral theory. In particular it is essential to develop a framework in