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Stochastic integration and differential equations—a new approach, by Philip Protter. Springer-Verlag, Berlin and New York, 1990, 302 pp., $48.00. ISBN-0-387-50996-8

Stochastic integration and stochastic differential equations are important for a wide variety of applications in the physical, biological, and social sciences. In particular, the last decade has seen an explosion in applications to financial economics. The need for a theory of *stochastic* integration is readily seen by considering integrals of the form $\int_{[0,t]} X_s\, dM_s$ and noting that these can be defined path-by-path in a Stieltjes sense for all continuous integrands X only if the paths of M are locally of finite variation. This immediately precludes such important processes as Brownian motion and all continuous martingales as integrators, as well as many discontinuous martingales. Consequently, for a large class of martingales M, one must resort to a truly probabilistic or stochastic definition of such integrals. The origins of the theory of stochastic integration lie in the early work of Wiener and the seminal work of Itô [11], where integrals with respect to Brownian motion were defined. Most importantly for applications, Itô developed a change of variables formula for C^2 functions of Brownian motion. In presenting the results of Itô in his book [7], Doob recognized that the two critical properties of Brownian motion B used in Itô's development of the stochastic integral were that B and $\{B_t^2 - t, t \geq 0\}$ are martingales. Extrapolating from this, Doob proposed a general integral with respect to L^2-martingales, which hinged on an as yet unproved decomposition theorem for the square of an L^2-martingale. This is a special case of a decomposition theorem for submartingales (the Doob-Meyer decomposition theorem), which was subsequently proved by Meyer [17, 18]. Using this decomposition result, Kunita and Watanabe

[14] made the next significant step in developing the stochastic integral, and an attendant change of variables formula, for a large class of square integrable martingales, including the continuous ones. Following the work of Kunita and Watanabe, Meyer and Doleans-Dade [19, 6] extended the definition of the stochastic integral and the change of variables formula to all local martingales and subsequently to semimartingales. It was in these works that the confinement to predictable integrands was seen to be essential [19], and the restriction, inherited from Markov process theory, that the filtrations be quasi-left continuous was removed [6]. The extension from square integrable martingales to local martingales (and hence to semimartingales) was later greatly simplified by the fundamental theorem for local martingales, which is due to Jia-an Yan [21] and Catherine Doléans-Dade [6A], independently. The natural role of semimartingales in the theory of stochastic integration was made clear by the discovery of Bichteler [1, 2] and Dellacherie [4], that semimartingales are the most general class of integrators for which one can have a reasonable definition of a stochastic integral against predictable integrands. Inspired by the survey article of Dellacherie [4], Protter has taken this as his starting point for a novel approach to stochastic integration. His book contains many results that he has either developed himself or given alternative proofs. To facilitate comparison of this alternative approach with the more conventional one, a summary of one version of the conventional approach is given below. The reader seeking a more cryptic analysis of Protter's book is free to turn to the last paragraph.

We take as given a complete probability space $(\Omega, \mathscr{F}, P)$, together with a filtration $\{\mathscr{F}_t\}$, i.e., an increasing family of sub-σ-fields of $\mathscr{F}$, which is assumed to satisfy the usual conditions of right continuity and inclusion of all P-null sets. A stochastic process is a function $Z: \mathbf{R}_+ \times \Omega \to \mathbf{R}$ such that for each $t \geq 0$, $Z_t = Z(t, \cdot)$ is a measurable function from $(\Omega, \mathscr{F})$ into $\mathbf{R}$ (with the Borel σ-field); Z is said to be *adapted* (to $\{\mathscr{F}_t\}$) if for each t, Z_t is a measurable function from $(\Omega, \mathscr{F}_t)$ into $\mathbf{R}$; Z is said to be *cadlag* (after the French: continu à droite limité à gauche) if its sample paths $\{t \to Z_t(\omega), \omega \in \Omega\}$ are right continuous with finite left limits. All (local) martingales will be assumed to have cadlag paths (in fact, there always exists a version with this property). A *semimartingale* is a cadlag, adapted stochastic process that can be decomposed as the sum of a (cadlag) local martingale and a cadlag, adapted process whose paths are locally of finite variation. Since one can make sense of integrals with respect to processes of the latter type in a path-by-path Stieltjes sense, to define stochastic integrals with respect to semimartingales, it suffices to define integrals with respect to local martingales. In fact, by the fundamental theorem for local martingales [21], any local martingale can be decomposed as the sum of a local L^2-martingale and a process whose paths are locally of finite variation. Thus, it suffices to define stochastic integrals with respect to local L^2-martingales, and by localization, one can further reduce to L^2-martingales. An important, desirable property for such integrals is that for an L^2-martingale M and bounded integrand X, the integrals $\{\int_{[0,t]} X_s\, dM_s,\ t \geq 0\}$ define an L^2-martingale. That is, the L^2-martingale property should be preserved by the stochastic integral. The general class of integrands X for which this holds for arbitrary L^2-martingale integrators (in particular, for those with discontinuities), is the class of predictable processes. The *predictable* σ-field $\mathscr{P}$ on $\mathbf{R}_+ \times \Omega$ is the σ-field generated by the *left* continuous, adapted stochastic processes. Alternatively, the predictable σ-field is generated by the sets of the form $(s, t] \times F_s$, for $F_s \in \mathscr{F}_s$, $0 \leq s \leq t < \infty$, together with the sets of the form $\{0\} \times F_0$, for $F_0 \in \mathscr{F}_0$. For an L^2-martingale M, the stochastic integral $\int X\, dM = \int_{[0,\infty)} X\, dM$ is first defined for simple predictable integrands X of the form:

$$(1) \quad X(t, \omega) = c_0 1_{\{0\}\times F_0}(t, \omega) + \sum_{i=1}^{n} c_i 1_{(s_i, t_i]\times F_i}(t, \omega), \qquad (t, \omega) \in \mathbf{R}_+ \times \Omega,$$

where $c_0 \in \mathbf{R}$, $F_0 \in \mathscr{F}_0$, $c_i \in \mathbf{R}$, $0 \leq s_i < t_i < \infty$, and $F_i \in \mathscr{F}_{s_i}$, for $1 \leq i \leq n$; in which case,

$$(2) \qquad \int X\, dM \equiv \sum_{i=1}^{n} c_i 1_{F_i}(M_{t_i} - M_{s_i}).$$

It can be shown that there is a unique σ-finite measure μ_M on $(\mathbf{R}_+ \times \Omega, \mathscr{P})$ such that

$$\mu_M((s, t] \times F) = E(1_F(M_t - M_s)^2)$$

for all $0 \leq s < t < \infty$ and $F \in \mathscr{F}_s$, and

$$\mu_M(\{0\} \times F_0) = 0,$$

for all $F_0 \in \mathscr{F}_0$. Indeed, for all $A \in \mathscr{P}$,

$$\mu_M(A) = E\left(\int_{[0,\infty)} 1_A(s, \omega) d[M, M]_s\right),$$

where $[M, M]$ is the quadratic variation process of M, which is the cadlag, increasing, adapted process such that for each t,

$$[M, M]_t = \lim_{n\to\infty} \sum_i (M_{t^n_{i+1}} - M_{t^n_i})^2,$$

where the sum is over all i such that t^n_i, $t^n_{i+1} \in \pi_n$, and for each n, π_n is a partition $0 = t^n_0 < t^n_1 < \cdots < t^n_{k_n} = t$ of $[0, t]$ such that $|\pi_n| = \max_i |t^n_{i+1} - t^n_i| \to 0$ as $n \to \infty$. Then, by the orthogonality properties of the L^2-martingale M, $\int X\,dM$ as defined in (2) satisfies

$$E\left(\left(\int X\,dM\right)^2\right) = \int_{\mathbf{R}_+\times\Omega} X^2\,d\mu_M. \tag{3}$$

This isometry is the key to the extension of the stochastic integral to other predictable integrands. Indeed, since simple functions of the form (1) are dense in $\mathscr{L}^2 \equiv L^2(\mathbf{R}_+ \times \Omega, \mathscr{P}, \mu_M)$, one can extend the definition of the integral $\int X\,dM$ via the L^2-isometry (3), to all $X \in \mathscr{L}^2$. Then, for X such that $1_{[0,t]}X \equiv 1_{[0,t]\times\Omega}X \in \mathscr{L}^2$ for each t, one can define

$$\int_{[0,t]} X_s\,dM_s = \int 1_{[0,t]}X\,dM.$$

It can be shown that this defines an L^2-martingale $Y = \{Y_t = \int_{[0,t]} X_s\,dM_s, t \geq 0\}$. One can extend the integral to local L^2-martingale integrators and suitable predictable integrands, using a localization procedure via stopping times; and then one can finally extend to semimartingales as integrators. A change of variables formula for C^2 functions of semimartingales can then be derived. The quadratic variation process $[Z, Z]$ for a semimartingale Z is defined in an analogous manner to that for an L^2-martingale. The path-by-path continuous part of this cadlag, increasing process is denoted by $[Z, Z]^c$. In particular, one has

$$[Z, Z]_t = [Z, Z]^c_t + \sum_{0<s\leq t} (\Delta Z_s)^2,$$

where ΔZ_s denotes the jump of Z at s. We can now state the change of variables formula for a semimartingale Z and a function $f \in C^2(\mathbf{R})$: the process $f(Z)$ is a semimartingale and the following formula holds,

(4)
$$f(Z_t) - f(Z_0) = \int_{[0,t]} f'(Z_{s-})\,dZ_s + \frac{1}{2}\int_{[0,t]} f''(Z_{s-})d[Z, Z]^c_s + \sum_{0<s\leq t}\{f(Z_s) - f(Z_{s-}) - f'(Z_{s-})\Delta Z_s\}.$$

Here Z_{s-} denotes the left limit of Z at s (defined to equal Z_s for $s = 0$), and the first integral in (4) is a stochastic integral whereas the second is an ordinary Stieltjes integral. Note that the last term only comes into play when Z has jumps (i.e., is discontinuous). The first line of (4) thus represents the simplified change of variables formula for the case when Z is continuous (in this case, $[Z, Z] = [Z, Z]^c$). Here one clearly sees how stochastic calculus differs from ordinary Newton calculus with the addition of the integral with respect to $[Z, Z]^c$, which comes from the unbounded variation of the continuous local martingale part of Z. A multidimensional version of (4) for n-tuples of semimartingales can also be derived. Here the polarization of $[Z, Z]^c$ for a pair of semimartingales comes into play. For the remainder of this review, to distinguish them from the semimartingales defined by Protter, we shall refer to semimartingales as defined above as *classical* semimartingales. We also continue to view $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ as the ambient filtered probability space on which all stochastic processes are defined.

In his approach to stochastic integration, Protter takes as primitive, simple predictable integrands of the form

$$X(t, \omega) = H_0(\omega)1_{\{0\}}(t) + \sum_{i=1}^{n} H_i(\omega)1_{(T_i(\omega), T_{i+1}(\omega)]}(t) \quad \text{for } (t, \omega) \in \mathbf{R}_+ \times \Omega,$$

where $H_0 \in \mathcal{F}_0$, $0 \leq T_1 \leq T_2 \leq \cdots \leq T_n < \infty$ are finite-valued stopping times, and $H_i \in \mathcal{F}_{T_i}$ with $|H_i| < \infty$ for $1 \leq i \leq n$. Denoting the collection of all such X by $\mathbf{S}$, we endow it with the topology of uniform convergence on $\mathbf{R}_+ \times \Omega$. Let $\mathbf{L}^0$ denote the space of all real-valued random variables on $(\Omega, \mathcal{F}, P)$, with the topology induced by convergence in probability. For a given stochastic process Z, define

$$I_Z(X) = \sum_{i=1}^{n} H_i(Z_{T_{i+1}} - Z_{T_i}). \tag{5}$$

(Protter also includes a term H_0Z_0 in his definition of $I_Z(X)$. This integral corresponds to an implicit assumption that $Z_{0-} = 0$ and consequently that Z has a jump from zero to Z_0 at time zero. The advantage of this is not clear. In fact, it is misleading when Z is continuous and leads to a more cautious statement of the change of variables formula. Since there is not common agreement on the correct convention, the choice that is consistent with the preceding paragraph has been elected here. Another way of achieving agreement between the two definitions would be to simply require that $Z_0 = 0$.) Protter defines a semimartingale to be a cadlag, adapted process Z such that for each $t \geq 0$, the mapping $I_{Z(\cdot\wedge t)}: \mathbf{S} \to \mathbf{L}^0$ is continuous. It can be readily verified that each local L^2-martingale is a semimartingale, and so is each cadlag, adapted process that is locally of finite variation. This allows one to show that Brownian motion, as well as Lévy process, are semimartingales. Using his definition of a semimartingale, Protter is able to give simple proofs of some results in stochastic integration that are nontrivial to prove when the classical definition of a semimartingale is used. For example, he gives a simple proof of Stricker's theorem: if Z is a semimartingale with respect to the filtration $\mathscr{F} = \{\mathscr{F}_t\}$, and $\mathscr{G} = \{\mathscr{G}_t\}$ is a subfiltration of $\mathscr{F}$ such that Z is adapted to $\mathscr{G}$, then Z is a semimartingale with respect to $\mathscr{G}$. Given a semimartingale Z, one can define a stochastic integral process $J_Z(X) = \{J_Z(X)(t), t \geq 0\}$, in the same way that $\int_{[0,t]} X_s\, dM_s$ was defined from $\int X\, dM$:

$$J_Z(X) = \sum_{i=1}^{n} H_i(Z_{\cdot\wedge T_{i+1}} - Z_{\cdot\wedge T_i}).$$

Let $\mathbf{L}$ denote the space of all adapted, caglad (left continuous with finite right limits) processes, and $\mathbf{D}$ the space of all adapted, cadlag processes. Note that $\mathbf{S} \subset \mathbf{L}$ and the stochastic integral process $J_Z(X)$ is in $\mathbf{D}$. Because of the localization to the interval $[0, t]$ that is implicit in the definition of $J_Z(X)(t)$, we need to define new topologies on $\mathbf{L}$ and $\mathbf{D}$ before extending the definition of $J_Z(X)$ to all $X \in \mathbf{L}$. A sequence of processes $\{X^n\}$ is said to converge to a process X *uniformly on compacts in probability* (ucp) if for each $t \geq 0$,

$$\sup_{0 \leq s \leq t} |X_s^n - X_s| \to 0 \quad \text{in probability as } n \to \infty.$$

Now, **S** is dense in **L** with the ucp topology, **D** with the ucp topology is metrizable as a complete metric space, and the linear mapping $J_Z : \mathbf{S} \to \mathbf{D}$ is continuous when $\mathbf{S} \subset \mathbf{L}$ and **D** are endowed with their ucp topologies. It follows that J_Z can be extended to a continuous linear mapping from **L** into **D**. This then defines the stochastic integral $\int_{[0,t]} X_s \, dZ_s \equiv J_Z(X)(t)$ of $X \in \mathbf{L}$ with respect to Z on $[0, t]$, for each $t \geq 0$. The class of integrands **L** suffices for the purposes of deriving the change of variables formula (4) and for the many applications that derive from it. As examples, Protter derives the formula for the stochastic exponential, i.e., the unique semimartingale solution Y of the stochastic equation

$$Y_t = 1 + \int_{[0,t]} Y_{s-} \, dZ_s, \qquad t \geq 0,$$

which is

$$Y_t = \exp\left(Z_t - \frac{1}{2}[Z, Z]_t\right) \prod_{0<s\leq t} (1 + \Delta Z_s) \exp\left(-\Delta Z_s + \frac{1}{2}(\Delta Z_s)^2\right).$$

Other applications he gives are a derivation of Lévy's characterization of Brownian motion, and Lévy's stochastic area formula. However, the space **L** is not sufficiently large for the treatment of some important topics such as local time and the martingale representation theorem, the latter being crucial for some recent applications of stochastic calculus in financial economics. To extend the definition of the stochastic integral to a larger class of predictable integrands, Protter needs to make connection with the conventional theory. In Chapter 3, he gives elementary proofs of some of the deep theorems in classical (semi)martingale theory, including the Doob-Meyer decomposition theorem, the fundamental theorem for local martingales, and the Girsanov change of measure formula, which culminate in a proof of the Bichteler-Dellacherie theorem that shows a classical semimartingale is the same as a semimartingale. In this exposition, Protter resurrects the notion of a natural process, which has all but disappeared from the modern literature, being replaced by the notion of a predictable process. (These two notions, as defined by Protter, are equivalent for bounded finite variation processes.) Having shown the equivalence of the two notions of semimartingale in Chapter 3, in the next chapter Protter is able to extend the definition of the stochastic integral to the most general class of predictable integrands, and to give applications to local time and the martingale representation theorem. A welcome last section in Chapter 4 treats Azema's martingale—a subject not often found in other texts. The final chapter (5), is devoted to stochastic differential equations. Besides establishing existence and uniqueness under Lipschitz conditions on the coefficients, Protter discusses stability of stochastic differential equations, Stratonovitch equations, and stochastic flows. He proves the Markov property for stochastic differential equations driven by Lévy processes (including Brownian motion) using results established in the previous chapter on the dependence of stochastic integrals on a parameter; this affords an elegant proof of a delicate measurability property which is often overlooked by less careful authors. The chapter on stochastic differential equations is quite lengthy and technical in parts. It is most likely that it will be used as a reference by applied researchers interested in stability, and by stochastic differential geometers interested in stochastic flows.

In reading the text, this reviewer encountered occasional items that would prove frustrating to the diligent student. Several of these are listed here for the convenience of the reader. The term square integrable martingale is used frequently before its first definition on p. 147. Although this term has wide acceptance in the theoretical martingale community, the uninitiated might confuse these martingales with those that are square integrable at each time t. At the bottom of p. 34, the author states that a locally square integrable local martingale is a locally square integrable martingale. Taken literally this implies that a locally square integrable local martingale is a martingale, which is certainly not true. In Chapter 3, the author defines the term natural for a finite variation process of integrable variation, and then in some subsequent uses of the term (and its local analogue) neglects to proclaim that the process is of integrable variation. Presumably, the author means this latter property to be part of the definition of natural in such circumstances. The introduction to Lévy processes given in §I.4 is very welcome; such a summary being difficult to find in any other text. The reader should note however that this material has not been vetted for minor errors as well as many other sections of the book. For example, in the proof of Theorem 30, the set Λ is used with at least two different meanings, without warning of this. Throughout the text, there are a number of cross-references to theorems that are off by some random integer. Some more applications for Lévy processes in the later parts of the text would be a good addition to a revised edition.

To summarize, there are by now a variety of books on stochastic integration and its applications. These range from those presenting the abstract theory, e.g., [5, 12, 16], through those containing a mix of the general (discontinuous) theory with applications, e.g., [23], to those that largely confine themselves to the simpler, albeit very important, case of continuous martingales or even Brownian motion, e.g., [3, 8, 9, 10, 13, 15, 22]. A common feature of all of these texts is that they adopt the conventional approach to stochastic integration of first defining stochastic integrals with respect to L^2 (or even L^2-bounded) martingales, before extending to local martingales and semimartingales. Following the model presented in Dellacherie [4], Protter has taken the Bichteler-Dellacherie theorem as starting point and defined semimartingales as "good" integrators acting on simple predictable integrands. One weakness with his approach is that although Protter gives a reason as to why one might want to choose adapted processes as integrands, he does not give a rationale for why one should single out the predictable ones, rather than say the optional ones. Of course, in the conventional theory, the desire to have L^2-martingales yield stochastic integrals that are L^2-martingales provides a very clear rationale. This (anticipating) point aside, Protter's approach allows him to very quickly define stochastic integrals with respect to left continuous, adapted processes, and to prove the change of variables formula for semimartingales. This suffices for a wide variety of applications, and for such it could be argued that Protter's approach provides a more streamlined entrée to them. However, for other important applications, it is necessary to make the connection with the conventional theory, as is done in Chapter 3 with an elementary development of the Bichteler-Dellacherie theorem. From this point on, it is doubtful that one could argue that one of the approaches, either the conventional one or the novel approach presented in this book, is more efficient. The author does however round out the book with two Chapters (4 and 5) on various important topics such as local time for semimartingales, the martingale representation theorem, and stochastic differential equations. The applications given in Protter's book have some overlap with those in other texts, but the selection of material is not duplicated in any one text. The style and choice of applications is definitely influenced by the French, with an attendant striving for generality and beauty. This book fills the need for an accessible account of the general theory of stochastic integration and for a detailed exposition of such topics as the stochastic exponential, local time for semimartingales, the martingale representation theory, Azema's martingale, and stochastic differential equations, in generality. I see this book as lying between [22 and 12]. It could be used for a second graduate course in probability, with desirable prerequisites being a knowledge of martingale theory and some stochastic processes. It should be particularly useful for probability students interested in the general theory of stochastic integration, and for students from other fields such as economics and electrical engineering who may have an interest in applications that require the full generality of the discontinuous martingale stochastic integral.

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Uniform Fréchet algebras, by H. Goldmann. North-Holland, Amsterdam, 1990, 355 pp., $102.50. ISBN 0-444-88488-2

Uniform Fréchet algebras are a class of topological algebras modeled on the algebra $\mathrm{Hol}(\Omega)$ of all holomorphic functions on a given plane domain Ω. In the topology of uniform convergence on compact subsets of Ω, $\mathrm{Hol}(\Omega)$ is a complete metrizable topological algebra.

A *Fréchet algebra* (F-algebra) is a commutative topological algebra which is complete metrizable and which has a neighborhood basis of 0 consisting of multiplicative and convex sets. Let X be a locally compact Hausdorff space and let $C(X)$ be the algebra of all complex-valued continuous functions on X. We give $C(X)$