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Uniform Fréchet algebras, by H. Goldmann. North-Holland, Amsterdam, 1990, 355 pp., $102.50. ISBN 0-444-88488-2

Uniform Fréchet algebras are a class of topological algebras modeled on the algebra $\mathrm{Hol}(\Omega)$ of all holomorphic functions on a given plane domain Ω. In the topology of uniform convergence on compact subsets of Ω, $\mathrm{Hol}(\Omega)$ is a complete metrizable topological algebra.

A *Fréchet algebra* (F-algebra) is a commutative topological algebra which is complete metrizable and which has a neighborhood basis of 0 consisting of multiplicative and convex sets. Let X be a locally compact Hausdorff space and let $C(X)$ be the algebra of all complex-valued continuous functions on X. We give $C(X)$

the compact open topology. $C(X)$ will be an F-algebra provided that X has a countable exhaustion $\cdots K_n \subset K_{n+1} \cdots$ by compact sets such that for each compact set $K \subset X$, $K \subset K_n$ for some n, and X is first countable.

A *uniform Fréchet algebra* (uF-algebra) is a complete subalgebra of $C(X)$ which contains the constants and separates the points of X. Examples are (i) $C(X)$ itself, (ii) Hol(X) where X is a Stein manifold. If X is compact, $C(X)$ and its closed subalgebras are Banach algebras in the sup-norm on X.

Such Banach algebra, the so-called *uniform algebras*, have been studied since the 1950's, and their properties are treated in the books [Br, Ga, Le, St] etc. One objective of the book under review is to extend the study of uniform algebras to the case when the underlying space X is locally compact.

If A is a uniform algebra on the compact space X, one imbeds X in the space of all nonzero homomorphisms: $A \to \mathbf{C}$, the "spectrum of A," $M(A)$. Gelfand showed that $M(A)$ has a compact Hausdorff topology such that X is a closed subset of $M(A)$ and the elements of A are continuous functions on $M(A)$. Restricted to $M(A)\backslash X$, these functions behave like analytic functions in many ways. In particular, they obey a local maximum modulus principle. This suggests the possibility that $M(A)\backslash X$ may have "analytic structure," i.e. that there exist nonconstant maps $\lambda \to \phi(\lambda)$ from a domain in the λ-plane into $M(A)\backslash X$ such that the composition $f(\phi(\lambda))$ is analytic in λ for each f in A.

The author carries this search for analytic structure to the spectrum of a uF-algebra A, defined as the set of all nonzero continuous homomorphisms: $A \to \mathbf{C}$. In his words "the main part of this book is devoted to the problem when a given uF-algebra is topologically and algebraically isomorphic to Hol(X), X a suitable complex space. We obtain a function algebra characterization of certain classes of algebras of holomorphic functions. In particular we give various characterizations of Hol(X) in the case that X is—an n-dimensional Stein space, n-dimensional Stein manifold,—a Riemann domain over $\mathbf{C}^n$, a domain of holomorphy in $\mathbf{C}^n$... These results have been mainly obtained by Arens, Brooks, Carpenter and Kramm."

The reader may have noticed that in the definition of spectrum we demanded continuity of the homomorphism in the case of uF-algebras, but not for uniform algebras. The reason is "Michael's problem," a question raised by E. Michael in 1952: is every homomorphism of a Fréchet algebra into $\mathbf{C}$ continuous? The answer is not known in general, and is known to be Yes in many special cases. The author devotes a chapter to this problem, including a discussion of an interesting approach to it by Dixon and Esterle which reduces the problem to a question about holomorphic self-mappings of $\mathbf{C}^p$. The author points out that one starting point of the study of Fréchet algebras are the papers of Arens [Ar] and Michael [Mi].

Part 3 of the book, Chapters 11 through 19, concerns analytic structure in the spectrum of a uF-algebra. An algebra A of continuous functions on a Hausdorff space Y is called a *maximum modulus algebra on* Y if for each compact set K in Y and f in A we have $\max|f|$ over $K = \max|f|$ over ∂K. In particular, Y has no compact subset with empty boundary. The subject of maximum modulus algebras originated in the beautiful paper [Ru] by Walter Rudin in 1953. The following result is given by Goldmann, [Go1]:

Theorem. *Let A be a uF-algebra on a locally compact and connected space X and let $f_1, \dots, f_n$ be an n-tuple of elements of A. Denote by F the map: $X \to \mathbf{C}^n$ with $F(x) = (f_1(x), \dots, f_n(x))$. For each complex line L in $\mathbf{C}^n$ which meets $F(X)$, put $X_L = F^{-1}(L)$. Assume that F is locally injective and further assume that the restriction algebra $A|_{X_L}$ is a maximum modulus algebra on X_L for each such complex line L. Then F is an open mapping and X can be equipped with the uniquely determined structure of a Riemann domain over $\mathbf{C}^n$ such that $A \subset \mathrm{Hol}(X)$. If, moreover, X is the spectrum of A, then X is Stein and $A = \mathrm{Hol}(X)$.*

A result of this type was given by E. Bishop in [Bi] for the case that A is a uniform algebra and $n = 1$. Related results by Rusek, Basener, Sibony and Kumagai are treated.

Not all maximum modulus algebras are algebras of holomorphic functions on complex spaces. If X is the singularity set of an analytic function in $\mathbf{C}^2$, then the algebra of polynomials on $\mathbf{C}^2$, restricted to X, is a maximum modulus algebra on X. This was shown by the reviewer [We] and by Slodkowski [S1]. Such an X can be a pretty terrible set. Goldmann gives an example of a uF-algebra A with the maximum modulus property such that the spectrum of A contains no analytic disk, yet the "identity theorem" of function theory holds: if f is in A and f vanishes on an open subset of the spectrum, then f vanishes identically [Go2].

SPECIAL TOPICS

(1) If A is a Banach algebra and $f_1, \ldots, f_r$ are elements of A having no common zero in the spectrum of A, then there exist $g_1, \ldots, g_r$ in A with $\sum_{i=1}^{r} f_i g_i = 1$.

The corresponding result is true for F-algebras, but is more difficult to prove. It was proved by Arens. The following generalization, due to Brooks, [Bro], is given in Chapter 6: Let A be an F-algebra and let $\{f_n | n = 1, 2, \ldots\}$ be an infinite sequence of elements of A having no common zero in the spectrum. Then there exists $\{g_n\}$ in A such that $\sum_{i=1}^{\infty} f_i g_i = 1$.

(2) A commutative semisimple Banach algebra A has zero as its only derivation into itself, where the map $D: A \to A$ is a *derivation* if it is linear and $D(ab) = aD(b) + bD(a)$ for all a, b in A. Obviously the corresponding result is false for F-algebras: The map $f \to f'$ is a nonzero derivation of $\mathrm{Hol}(\Omega)$ into itself, for Ω a plane domain. In Chapter 8, the result of Carpenter, [Ca], is given: Every derivation of a semisimple F-algebra into itself is continuous. The proof is based on a result of B. E. Johnson.

(3) There are issues of pointset topology which arise, both on the relation between the topology of the underlying space X and the nature of the topological algebra $C(X)$, and on the topology of the spectrum of an F-algebra. Such questions are treated in Chapter 7.

(4) There are connections with the study of local rings, treated in Chapter 17.

Helmut Goldmann deserves thanks for having written a well-organized and thought-provoking book covering a large range of interesting questions in functional analysis.

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Holomorphic Hilbert modular forms, by Paul B. Garrett.Wadsworth & Brooks/Cole, 1990, 304 pp., ISBN 0-534-10344-8

In the early 1970s many mathematicians, especially number theorists, learned that they were secretly in love with $GL(2)$. This circumstance was brought to light in large part by the publication in 1970 of the book *Automorphic forms on* $GL(2)$ by H. Jacquet and R. P. Langlands [JL]. The last year has seen the publication of no fewer than three introductory books, with completely different tables of contents, on the subject of Jacquet-Langlands' formidable monograph: *Modular forms* by T. Miyake, *Hilbert modular forms* by E. Freitag, and the book under review. Of the three, Miyake's book, which treats only the case of $GL(2, \mathbf{Q})$, is closest in content, if not in spirit, to Jacquet-Langlands; Freitag and Garrett cover much of the same ground but have quite different destinations in mind.