BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 25, Number 1, July 1991
©1991 American Mathematical Society
0273-0979/91 $1.00 + $.25 per page

Introduction to holomorphic functions of several variables, by Robert C. Gunning. Wadsworth & Brooks/Cole, vol. 1, *Function theory*, 202 pp. ISBN 0-534-13308-8; vol. 2, *Local theory*, 215 pp. ISBN 0-534-13309-6; vol. 3, *Homological theory*, 194 pp. ISBN 0-534-13310-X

What is several complex variables? The answer depends on whom you ask.

Some will speak of coherent analytic sheaves, commutative algebra, and Cousin problems (see for instance [GRR1, GRR2]).

Some will speak of Kähler manifolds and uniformization—how many topologically trivial complex manifolds are there of strictly negative (bounded from zero) curvature (see [KOW])? (In one complex variable the important answer is *one*.)

Some will speak of positive line bundles (see [GRA]).

Some will speak of partial differential equations, subelliptic estimates, and noncoercive boundary value problems (see [FOK]).

Some will speak of intersection theory, orders of contact of varieties with real manifolds, and algebraic geometry (see [JPDA]).

Some will speak of geometric function theory (see [KRA]).

Some will speak of integral operators and harmonic analysis (see [STE1, STE2, KRA]).

Some will speak of questions of hard analysis inspired by results in one complex variable (see [RUD1, RUD2]).

As with any lively and diverse field, there are many points of view, and many different tools that may be used to obtain useful results. Several complex variables, perhaps more than most fields, seems to be a meeting ground for an especially rich array of techniques. Thus when one writes a book on this subject there are a number of difficult choices to make.

In my view the first book in the "modern era" of several complex variables was that of Gunning and Rossi [GUR]. To be sure, there were other books (c.f. [OSG, BOM, HER, BET])—and important research—before this one, but this is the book that introduced most of us currently working in the field to the subject of several complex variables. It was comprehensive, detailed, and it set the tone of the subject for many years. And the authors found a focus for their project by making the book *algebraic* in orientation.

At this point it is appropriate to be more discursive about what we mean when we speak of orientations or approaches in the subject of several complex variables. We begin with some definitions. A *domain* in $\mathbf{C}^n$ is a connected open set. A domain Ω is called a *domain of holomorphy* if there is a holomorphic function on Ω that cannot be analytically continued to any larger open set. The first big problem in this subject is to identify, in an intrinsic geometric fashion, the domains of holomorphy. (Note in passing that in one complex variable *every* open set is a domain of holomorphy—this is an exercise using the Weierstrass theorem, or see [KRA].)

It is not difficult to see that any convex set is a domain of holomorphy. Assume for simplicity that Ω is a convex domain with C^2 boundary. Then there is a C^1 function $\rho : \mathbf{C}^n \to \mathbf{R}$ such that

$$\Omega = \{z \in \mathbf{C}^n : \rho(z) < 0\}$$

and $\nabla\rho \neq 0$ on $\partial\Omega$. (The existence of such a ρ follows from the Implicit Function Theorem.) If $P \in \partial\Omega$ then the function

$$\Phi_P(z) = \frac{1}{\sum_{j=1}^{n} \frac{\partial\rho}{\partial z_j}(P)(z_j - P_j)}$$

is holomorphic on Ω; but it blows up at P hence cannot be analytically continued past P. It is not difficult to see, using an infinite product construction, that from the functions Φ_P can be constructed a function holomorphic on Ω that cannot be analytically continued past any boundary point. Thus our convex domain is a domain of holomorphy.

As the Riemann mapping theorem shows, the property of convexity is not a holomorphic invariant. Thus one knows on a priori grounds that the convex domains cannot comprise all the domains of holomorphy. E. E. Levi and F. Hartogs ascertained the correct invariant condition generalizing the classical notion of convexity. First, we call a function u on a domain $\Omega \subseteq \mathbf{C}^n$ *plurisubharmonic* if it is upper semi-continuous and if the restriction of u to the intersection of Ω with any complex line through Ω is subharmonic. A real-valued function Ψ on Ω is called an *exhaustion function* for Ω if for any $c \in \mathbf{R}$ the set $\{z \in \Psi : \Phi(z) \leq c\}$ is relatively compact in Ω. Finally, Ω is said to be *pseudoconvex* if it possesses a plurisubharmonic exhaustion function.

It is difficult to grasp the geometric content of the notion of pseudoconvexity. Suffice it to say that it is a holomorphically invariant version of the classical notion of convexity. The book [KRA] fleshes out this last statement. Matters *will* become more geometric in a moment when we introduce a refined notion of pseudoconvexity called strong pseudoconvexity.

The *Levi problem* is to prove that the domains of holomorphy are precisely the pseudoconvex domains. The forward direction of this assertion, while not routine, is not very difficult and can be presented in a few lectures. This part of the problem was resolved by the French and German schools prior to World War II. The implication that a pseudoconvex domain is a domain of holomorphy is quite difficult and was not proved in full until the mid-1950s.

An important reduction in the Levi problem has two parts: (i) that any pseudoconvex domain can be exhausted by a sequence of much nicer domains called *strongly pseudoconvex* (more on these below); (ii) that the increasing union of domains of holomorphy is also a domain of holomorphy (see [BER]). Thus it suffices to prove the hard part of the Levi problem for strongly pseudoconvex domain. A domain Ω is said to be strongly pseudoconvex if each boundary point P has a neighborhood U_P and a holomorphic change of coordinates on U_P such that, in the new coordinates, $\partial\Omega \cap U_P$ is strongly convex (all boundary curvatures are positive). We saw in our discussion above that convex boundary points are not difficult to treat. If P is a boundary point of a strongly pseudoconvex domain and U_P is as above then, on $U_P \cap \Omega$, there is a holomorphic function F_P that blows up at P. Thus the Levi problem reduces to producing from F_P a holomorphic function *on all of* Ω that is singular at P. It is this last statement that Gunning isolates and calls "the Levi problem": to manufacture from locally defined holomorphic data a holomorphic function defined on the whole domain. In one complex variable we have both Weierstrass's theorem and the Mittag-Leffler theorem for performing this procedure. In several complex variables this question lies very deep.

The First Cousin Problem is perhaps the oldest technique formulated to study the "local-to-global" problem enunciated above. Suppose that Ω is covered by open sets $U_1, U_2, \ldots$ and that on each nonempty $U_i \cap U_j$ there is given a holomorphic function g_{ij}. We postulate that $g_{ij} = -g_{ji}$ and that if $U_i \cap U_j \cap U_k \neq \varnothing$ then $g_{ij} + g_{jk} + g_{ki} = 0$ on $U_i \cap U_j \cap U_k$. The First Cousin Problem is, given such a collection of g_{ij}, to find functions g_i on U_i, all i, such that $g_{ij} = g_j - g_i$ on $U_i \cap U_j$.

Suppose that Ω is a strongly pseudoconvex domain and that we know that the First Cousin Problem can always be solved on strongly pseudoconvex domains. Fix $P \in \partial\Omega$ and let U_P be a "convexifiable" neighborhood of P as in the definition of strong pseudoconvexity. A moment's thought shows that the concept of strong pseudoconvexity is a stable one. Thus there is a domain $\widehat{\Omega}$ that contains $\overline{\Omega}$ and is still strongly pseudoconvex. Set $U_1 = U_P$ and take U_2 to be another open set such that $U_1 \cup U_2 \supseteq \widehat{\Omega}$ and $P \notin \overline{U_2}$. We may assume that the locally defined singular function F_P is defined on $U_P \cap \widehat{\Omega} \cap U_2$. Then $U_1 \cap U_2 \neq \varnothing$ (else $U_1 \supseteq \Omega$ and there is nothing to do). Set $g_{12} = -F_P$ on $U_1 \cap U_2$. We solve the First Cousin Problem for this data on $\widehat{\Omega}$ and obtain holomorphic functions g_1 on U_1 and g_2 on U_2. Define

$$H_p(z) = \begin{cases} g_1(z) - F_P(z) & \text{if } z \in U_1 \cap \Omega, \\ g_2(z) & \text{if } z \in U_2 \cap \Omega. \end{cases}$$

A moment's thought shows that H_P is well defined and holomorphic on Ω. Moreover H_P blows up at P since $-F_P$ does while g_1 continues analytically past P. Thus H_P is a function holomorphic on all of Ω that is singular at P. That solves the Levi Problem.

The device of sheaf theory was devised to put the First Cousin Problem on a more natural footing. Saying that the First Cousin Problem is always solvable is equivalent to saying that the first cohomology class with coefficients in the sheaf of germs of holomorphic functions is zero. Cartan's Theorem B asserts this last statement and much more. We shall say no more about the sheaf theoretic approach in this review.

The partial differential equations approach to extending a locally defined holomorphic function to a global one has a different flavor. First we need some definitions. For $j = 1, \ldots, n$ and f a C^1 function we define

$$\frac{\partial}{\partial \overline{z}_j} f = \frac{1}{2}\left(\frac{\partial}{\partial x_j} + i\frac{\partial}{\partial y_j}\right) f.$$

The $\overline{\partial}$, or Cauchy-Riemann operator, on a C^1 function f is given by

$$\overline{\partial} f = \sum_{j=1}^{n} \frac{\partial f}{\partial \overline{z}_j} d\overline{z}_j.$$

Here $d\bar{z}_j = dx_j - i dy_j$. If $\alpha = \sum_{k=1}^n \alpha_k(z)\, d\bar{z}_k$ is a $(0, 1)$ form then we set

$$\bar{\partial}\alpha = \sum_{k=1}^{n}\sum_{j=1}^{n} \frac{\partial \alpha_k}{\partial \bar{z}_j}\, d\bar{z}_j \wedge d\bar{z}_k.$$

It is straightforward to check that if f is a C^2 function then $\bar{\partial}\bar{\partial} f = 0$.

Now if α is a given $(0, 1)$ form on Ω with C^1 coefficients and if we seek a function u such that $\bar{\partial} u = \alpha$ then an obvious necessary condition (in view of the preceding paragraph) for u to exist is that $\bar{\partial}\alpha = 0$ on Ω. It is a deep theorem (see [HOR1]) that if Ω is pseudoconvex then this condition is also sufficient for solvability. More precisely, if $\alpha = \sum_j \alpha_j\, d\bar{z}_j$ has C^1 coefficients on Ω pseudoconvex and if $\bar{\partial}\alpha = 0$ then there is a C^1 function u on Ω such that $\bar{\partial} u = \alpha$. If we assume that α has coefficients in $L^2(\Omega)$ then we may take u to be in $L^2(\Omega)$. Let us see how to exploit this last result to solve the Levi problem on Ω.

Let Ω be strongly pseudoconvex and fix $P \in \partial\Omega$. Begin with the locally defined singular function F_P on $U_P \cap \Omega$. Choose a positive integer k so large that $(F_P)^k \notin L^2(\Omega)$. (We use here the fact that, by construction, F_P blows up at the boundary at least at the rate of the reciprocal of the distance to P—see [KRA] for details.) Let ϕ be a C_c^∞ function on $\mathbf{C}^n$ with support in U_P and such that $\phi \equiv 1$ near P. Define $\alpha = (\bar{\partial}\phi) \cdot (F_P)^k$. Then α, understood to be identically 0 off U_P, is a well-defined form on all of Ω and $\bar{\partial}\alpha = 0$ on Ω. Moreover $\bar{\partial}\phi$ vanishes in a neighborhood of P so that α has L^2 coefficients on Ω. Our hypothesis yields a function u on Ω such that $u \in L^2(\Omega)$ and $\bar{\partial} u = \alpha$. But then the function $h_P = \phi \cdot (F_P)^k - u$ satisfies $\bar{\partial} h_P = 0$ on Ω. Unraveling the definitions, we then see that h_P satisfies the Cauchy-Riemann equations in each variable hence is holomorphic in each variable separately. But that is one of several different and equivalent definitions of holomorphic function of several complex variables. Notice further that, since $u \in L^2$ and $\alpha \notin L^2$, it follows that h_P still has a singularity at P. Thus h_P is a globally defined holomorphic function on Ω that is singular at P. As indicated above, the functions h_P may be amalgamated into a single holomorphic function on Ω that is singular at every boundary point. Therefore Ω is a domain of holomorphy.

It should be noted that R. M. Range [RAN] has found a method for solving the Levi problem that uses integral representation formulas. This is the first new method for attacking the problem that has been produced in some time.

Of course there are other approaches to the Levi problem that we shall not cover here. What we wish to stress is that the detailed presentation of any of these methods comprises most of a first course in several complex variables. And it is only *after* the Levi problem is solved that the subject *begins*. Thus the entry level to the field of several complex variables is rather high. Again, this makes it rather difficult to decide what to include in a first book on the subject. The book under review chooses to say little about the $\bar{\partial}$ problem and instead concentrates on Cousin problems, sheaf theory, and the like.

Since I have spent some time above describing the Levi problem, I should perhaps point out that Gunning's approach is somewhat different from that in most of the books in print. He first proves the Dolbeault isomorphism (that the cohomology coming from the $\bar{\partial}$ complex is isomorphic to the cohomology with coefficients in the sheaf of germs of holomorphic functions). Then he uses an induction on dimension to establish the Levi problem. This allows Gunning to prove the result quickly and cleanly and to get on with the more algebraic topics. For my taste this is not the best proof, for it suppresses the essential geometry of the Levi problem; but it suits Gunning's perspective and fits into the book well.

Speaking as an analyst, I would say that the good feature of [GUR] was its modernness (for its day) and its comprehensiveness. It was the first "post Levi problem" book. The negative feature of [GUR] (and I say this only platonically) is that it presents the subject as a polished gem, with nary a foothold for the neophyte looking for things to do. The book of Hörmander [HOR2], written from the diametrically opposite partial differential equations point of view (and at roughly the same time), has the same drawback. One is given a sparkling glimpse into a crystal ball. There are no rough edges, few calculations, little mention of open problems, no exercises.

But that was a different time. The books [GUR] and [HOR2] were intended to be monographs at the cutting edge of research. Written hard upon the resolution of the Levi problem (that pseudoconvex domains and domains of holomorphy are one and the same), their intent was to bring interested researchers of a very high calibre into the field. This was no time for textbooks.

Although [GUR] was the bellweather for the subject of several complex variables for many years, the subject developed a new vector (as they say at the National Science Foundation) soon after the book's appearance. The partial differential equations techniques of Kohn [KOH], developed while [GUR] was being written, began to reveal the rough edges whose absence was lamented above.[7] Here were Hilbert spaces, Fourier analysis, pseudodifferential operators (indeed the development of ψdo's was inspired in large part by the need for them in studying the $\bar{\partial}$-Neumann problem), and *estimates*. The advent of PDE techniques was followed in a few years by the nearly simultaneous development of integral formulas by Henkin, Ramirez, Kerzman, Grauert-Lieb, Øvrelid, and others. This particular development drew many classical one variable function theorists and *function algebraists* (see, for instance [WER]) to several complex variables. The work of the Stein school (see [STE1, STE2, FOS]) also attracted the attention of harmonic analysts.

There followed soon the "Princeton/Paris years" during which the example of Kohn-Nirenberg [KON] (of a nonconvexifiable pseudoconvex point) the example of Diederich-Fornaess [DIF2] (of a pseudoconvex domain without a neighborhood basis of pseudoconvex domains), the example of Bedford-Fornaess [BEF] (of a complex Monge-Ampère equation with smooth data but without smooth solution), the example of Sibony [SIB] (of a domain convex with respect to the family of *all* holomorphic functions but not with respect to the family of *bounded* holomorphic functions) as well as many new positive results revealed our aforementioned crystal ball to be more like a piece of volcanic rock (see [FRS] for a catalog of many of the most important examples). With the perspective of another fifteen years it seems clear that the statement of Cirka [CIR] was prophetic: "In the still young theory of ... holomorphic functions of several complex variables there is an entire field of white nothingness with widely spaced isolated results."

Around 1980 the dust was settling after the flurry of activity described in the preceding paragraph and it seemed time for some "texts." There appeared in rapid order the books of Rudin [RUD2], Range [RAN1], Field [FIE], Grauert-Fritzche [GRF],

[7] Although less revealing of rough edges, there also were important partial differential equations techniques developed around this time by Andreotti/Vesentini [ANV] and by Hörmander [HOR1].

Grauert-Remmert [GRR1, GRR2], and of the reviewer [KRA]. These books served, if nothing else, to validate the eclecticism of the field we have been describing. All of the viewpoints—from partial differential equations to algebraic, geometric, and function theoretic—are represented in the union of these books. It may be safe to say that by 1986 the textbook writing business was in stasis.

In 1971, while Halsey Royden was a dean at Stanford University, he proved his famous theorem to the effect that the Kobayashi metric on Teichmüller space and the Teichmüller metric coincide. He was fond of saying at the time that it was "the best theorem proved that year by a dean." The appearance of Gunning's three volume work in 1990 coincides almost exactly with his assuming the mantle of Dean of the Faculty at Princeton University. His book is a good candidate for the best written this year by a dean.

There is nobody better qualified than the author of the book being reviewed here to survey the sweep of activity that I have described and to put it into perspective. Gunning attacks the task in the large by writing three separate volumes. One is about function theory (culminating with the settling of the Levi problem), one is about local algebraic geometry, and one is about sheaf cohomology. It is clear by page count and level of detail that the author's heart is in the algebra volume, but all three are treated lovingly and elegantly.

It has been my opinion for some time now that many mathematics book are written in the wrong fashion. I mean this in the following precise sense. Many an author wants his book to be absolutely up to the moment. Given that with TEX the production process takes about four months and without TEX it can take over a year, this is an *a priori* impossibility. But never mind that. When an author sits down and revises his manuscript every time he gets a new preprint in the mail, his book will quickly lose its focus and, unfortunately, its audience. I think that this explains in part why most books in the math library exhibit a dog-eared first half and a pristine second half. What a book author *should* do is isolate a body of material and then devote himself to presenting it clearly. This serves the dual purpose of (i) helping the author keep the entire project in his view and (ii) helping the author to see the light at the end of the tunnel (so that he can finish the book). A better book results.

Gunning apparently wrote the book under review with the point of the previous paragraph well in mind. He describes a body of knowledge, as he sees it, from the perspective of 1990. But he only describes in any detail the developments prior to 1980. I think that this was absolutely the right decision. This is the right book at the right time. It will be a useful book for students, a good resource for researchers and, unlike previous books, it presents a panorama of much (but certainly not all) of the subject.

While as a scholar I find it convenient to have this book in three slim volumes, I must as a consumer question the decision of the publisher to release the book in that form. For it effectively triples the price of the book. Since mathematics books are already priced out of reach of many, this tends to aggravate an already difficult problem. On the other hand, I received my copy *gratis*.

In spite of my thrifty nature, I find real psychological advantage to three (approximately) two hundred page volumes over one six hundred page volume. A six hundred page mathematics book, whose climax may be on p. 598, is an onerous prospect for the reader. But how much trouble can we get into in two hundred pages? One even finds oneself, as with a paperback mystery, peeking at the last few pages to see whodunit. It is so easy to dip into these volumes—to pick a topic and read about *just that one item*—that it is clear that they will make the subject of several complex variables accessible to a broad audience.

This is a book that every worker in several complex variables, indeed every analyst, should have on his shelf. It describes a central, influential, and important area of mathematical analysis and will be the reference of choice for some time. The ten years of attention that Gunning lavished on this project shows on each page. The book will serve as a model of elegance and clarity for future writers in several complex variables.

References

[ANV] A. Andreotti and E. Vesentini, *Carleman estimates for the Laplace-Beltrami equations on complex manifolds*, Publ. Math. Inst. Hautes Études Sci. **25** (1965), 81–130.

[BEF] E. Bedford and J. E. Fornaess, *Counterexamples to regularity for the complex Monge-Ampère equation*, Invent. Math. **50** (1979), 129–134.

[BET] H. Behuke and P. Thullen, *Theorie der funktionen mehrerer komplexer veränderlichen*, Springer-Verlag, Berlin, 1934.

[BER] L. Bers, *Introduction to several complex variables*, New York Univ. Press, New York, 1964.

[BOM] S. Bochner and W. T. Martin, *Several complex variables*, Princeton Univ. Press, Princeton, NJ, 1948.

[CIR] E. Cirka, *The theorems of Lindelöf and Fatou in* $\mathbf{C}^n$, Mat. Sb. **92** (134) (1973), 622–644; Math. U.S.S.R. Sb. **21** (1973), 619–639.

[COG] M. Commichau and H. Grauert, *Das formale Princip für kompakte komplexe Untermannigfaltigkeiten mit* 1*-positive Normalenbüdel*, Recent Developments in Several Complex Variables (J. E. Fornaess, ed.), Ann. of Math. Studies, vol. 100, Princeton Univ. Press, Princeton, NJ, 1981.

[JPDA] J. D'Angelo, *Finite-type conditions for real hypersurfaces in* $\mathbf{C}^n$, (S. G. Krantz, ed.), Lecture Notes in Math., vol. 168, Springer-Verlag, Berlin, 1987, pp. 83–102.

[DIF1] K. Diederich and J. E. Fornaess, *Pseudoconvex domains: Bounded strictly plurisubharmonic exhaustion functions*, Invent. Math. **39** (1977), 129–141.

[DIF2] ——, *Pseudoconvex domains: An example with nontrivial Nebenhülle*, Math. Ann. **225** (1977), 275–292.

[FIE] M. Field, *Several complex variables and complex manifolds*, Cambridge Univ. Press, Cambridge, 1982.

[FOK] G. B. Folland and J. J. Kohn, *The Neumann problem for the Cauchy-Riemann complex*, Princeton Univ. Press, Princeton, NJ, 1972.

[FOS] G. B. Folland and E. M. Stein, *Estimates for the* $\overline{\partial}_b$ *complex and analysis of the Heisenberg group*, Comm. Pure Appl. Math. **27** (1974), 429–522.

[FRS] J. E. Fornaess and B. Stensones, *Lectures on counterexamples in several complex variables*, Princeton Univ. Press, Princeton, NJ, 1987.

[GRF] H. Grauert and K. Fritzsche, *Several complex variables*, Springer-Verlag, Berlin, 1976.

[GRL] H. Grauert and I. Lieb, *Das Ramirezsche Integral und die Lösung der Gleichung* $\overline{\partial}u = \alpha$ *im Bereich der beschränkten Formen*, Complex Analysis (Proc. Conf., Rice Univ., Houston, Tex., 1969), Rice Univ. Studies **56** (1970), no. 2, (1971), 29–50.

[GRR1] H. Grauert and R. Remmert, *Theory of Stein spaces*, Springer-Verlag, Berlin, 1979.

[GRR2] ——, *Coherent analytic sheaves*, Springer-Verlag, Berlin, 1984.

[GUR] R. C. Gunning and H. Rossi, *Analytic functions of several complex variables*, Prentice-Hall, Englewood Cliffs, NJ, 1965.

[HEN] G. M. Henkin, *Integral representations of functions holomorphic in strictly pseudoconvex domains and some applications*, Mat. Sb. **78** (120) (1969), 611–632; Math. U. S. S. R. Sb. **11** (1970), 273–281.

[HER] M. Hervé, *Several complex variables: Local theory*, Oxford Univ. Press, Oxford, 1963.

[HOR1] L. Hörmander, L^2 *estimates and existence theorems for the* $\overline{\partial}$ *operator*, Acta Math. **113** (1965), 89–152.

[HOR2] ——, *Introduction to complex analysis in several variables*, Van Nostrand, Princeton, NJ, 1967.

[KOW] S. Kobayashi and H. H. Wu, *Complex analytic geometry*, Birkhäuser, Boston, 1983.

[KOH] J. J. Kohn, *Harmonic integrals on strongly pseudoconvex manifolds*. I, Ann. of Math. (2) **78** (1963), 112–148; II, ibid. (2) **79** (1964), 450–472.

[KON] J. J. Kohn and L. Nirenberg, *A pseudo-convex domain not admitting a holomorphic support function*, Math. Ann. **201** (1973), 265–268.

[KRA] S. G. Krantz, *Function theory of several complex variables*, John Wiley and Sons, New York, 1982.

[OSG] W. F. Osgood, *Lehrbuch der Funktionenetheorie*, II, Teubner, Leipzig, 1924.

[OVR] N. Øvrelid, *Integral representation formulas and L^p estimates for the $\bar{\partial}$ equations*, Math. Scand. **29** (1971), 137–160.

[RAM] E. Ramirez, *Divisions problem in der komplexen analysis mit einer Anwendung auf Rand integral darstellung*, Math. Ann. **184** (1970), 172–187.

[RAN1] R. M. Range, *Holomorphic functions and integral representations in several complex variables*, Springer-Verlag, Berlin, 1986.

[RAN2] ——, *An elementary integral solution operator for the Cauchy-Riemann equations on pseudoconvex domains in* $\mathbf{C}^n$, Trans. Amer. Math. Soc. **274** (1982), 809–816.

[ROY] H. Royden, *Automorphisms and isometries of Teichmüller space*, Advances in the Theory of Riemann Surfaces (Proc. Conf., Stony Brook, NY, 1969), Annals of Math. Studies 66, Princeton Univ. Press, Princeton, NJ, 1971, pp. 369–383.

[RUD1] W. Rudin, *Function theory in polydiscs*, Benjamin, New York, 1969.

[RUD2] ——, *Function theory in the unit ball of* $\mathbf{C}^n$, Springer-Verlag, Berlin, 1980.

[SIB] N. Sibony, *Prolongement des fonctions holomorphies bornées et métrique de Carathéodory*, Invent. Math. **29** (1975), 205–230.

[STE1] E. M. Stein, *Boundary behavior of holomorphic functions of several complex variables*, Princeton Univ. Press, Princeton, NJ, 1972.

[STE2] ——, *Singular integrals and estimates for the Cauchy-Riemann equations*, Bull. Amer. Math. Soc. **79** (1973), 440–445.

[WER] J. Wermer, *Banach algebra and several complex variables*, Springer-Verlag, Berlin, 1976.

STEVEN G. KRANTZ
WASHINGTON UNIVERSITY

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 25, Number 1, July 1991
©1991 American Mathematical Society
0273-0979/91 $1.00 + $.25 per page

Model theoretic algebra, by Christian U. Jensen and Helmut Lenzing. Gordon & Breach, New York, 1989, 430 pp., $59.00. ISBN 2-88124-717-2

What is model-theoretic algebra? According to [4], it is the subject which "deals with algebraic structures or theories, aiming at model-theoretic results or using model-theoretic means." This definition does not strictly cover some important early papers where model-theoretic results and concepts were presented... without using model-theoretic means. The papers of Artin and