The reader will have surmised by now that the reviewer has a very high opinion of this book. The exposition is not perfect, with some repetition of arguments, and too many formulas, but such quibbles are overwhelmed by the service the authors have rendered by making such fascinating material available to a wide audience. The authors needed five years to write up their results; an outsider trying to learn all of this from partial results in the literature would have faced an impossible task. The book is accessible to any motivated reader, although it is not easy reading; it is essentially self-contained, except for a few results which are cited (with references) in the later chapters. It cannot be the definitive treatment, since the subject is just getting started; recent developments include Borcherds' "two-variable moonshine," with many new concepts and results, but it is impossible to imagine a future treatment of these matters which would not be heavily influenced by this book.

ANDREW OGG
UNIVERSITY OF CALIFORNIA, BERKELEY

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Group theory in physics, volume III, *Supersymmetries and infinite-dimensional algebras*, by J. F. Cornwell. Techniques of Physics (N. H. Marsh, ed.), Academic Press, New York, 1989, 615 pp., $55.00. ISBN 0-12-189805-9

The theory of classical Lie groups and algebras is of fundamental importance in mathematics as the meeting ground of algebra, analysis, and topology, and as an essential tool in modern physics. The book under review is the third in a series of volumes by Cornwell. In the first volume he covers some basic group theory, Lie groups, representation theory, and applications to molecular and solid state physics. In the second volume he discusses Lie algebras, their relationship with Lie groups, the structure theory of semisimple Lie algebras and their representation theory, Lorentz groups, Poincaré groups, and applications to the theory of elementary particles (global internal symmetries and gauge theory). Results and numbered equations from these two volumes are referred to in the third volume, making it a bit awkward to read without the others at hand. The third volume covers two significant generalizations of Lie theory, the Lie supergroups and superalgebras, and the infinite-dimensional Lie algebras (Kac-Moody algebras and the Virasoro algebra). These generalizations have come to play a large role in particle physics [GO] and involve serious new mathematics. The intention of the author is to overcome the communication barrier, which makes it difficult for some physicists to penetrate the style of pure mathematical exposition which is commonly used. This means that the book is written for an audience of physicists. For example, explicit bases are given for vector spaces, summations with many explicit indices are used instead of abstract descriptions, and representations are defined by giving the matrices which represent basis elements. There are no exercises, but there are five large appendices (147 pages) containing proofs, which were considered too distracting to include in the main text, facts about Clifford algebras, and tables of properties of the algebras studied in the book.

In the first part one is introduced to superalgebras, supermatrices, superspace, and supergroups, and given general properties of these algebras and their representations. A large section is devoted just to the Poincaré superalgebras, supergroups, and their representations. Familiarity is assumed with the theory of Clifford algebras and spinor representations, as presented in the appendix, and with the theory of Lorentz groups, Poincaré groups, and their representations, as presented in volume two of the series. Another section describes Poincaré supersymmetric fields in two formalisms. These are supersymmetric versions of quantum field theory, written for the reader who already has some familiarity with ordinary quantum field theory.

The next several sections are presented in a completely different style, much easier for a mathematical audience to read. They cover basic facts about the finite-dimensional simple Lie superalgebras, the infinite-dimensional Kac-Moody Lie algebras, their highest weight representations, vertex and spinor constructions of some representations, and the theory of the Virasoro algebra and its highest weight representations. The author successfully imparts a working knowledge of the essential results of these subjects without giving most proofs. He has made a serious effort to provide references to the literature where proofs and further details can be found. The book concludes with a section on the algebraic aspects of string and superstring theory, presented in the physicists style.

Despite the limitations of space in a review such as this, I would like to say something about some of the mathematical topics in this book. A Lie algebra is a vector space L equipped with a bilinear product "[,]", called the *bracket*, which is skew-symmetric, $[a, b] = -[b, a]$, and satisfies the Jacobi identity

$$[a, [b, c]] + [b, [c, a]] + [c, [a, b]] = 0.$$

A Lie superalgebra is a $\mathbb{Z}_2$-graded vector space $L = L_0 \oplus L_1$ equipped with a bilinear product called the superbracket, which satisfies graded versions of skew-symmetry and the Jacobi identity. Let $a \in L$ be called homogeneous if $a \in L_0$ or $a \in L_1$, and let $\deg(a) = i$ if $a \in L_i$. For a and b homogeneous elements of L let $\operatorname{sgn}(a, b) = (-1)^{\deg(a)\deg(b)}$. Then the graded version of skew-symmetry is $[a, b] = -\operatorname{sgn}(a, b)[b, a]$ and the graded Jacobi identity is

$$\operatorname{sgn}(a, c)[a, [b, c]] + \operatorname{sgn}(b, a)[b, [c, a]] + \operatorname{sgn}(c, b)[c, [a, b]] = 0.$$

Of course, this means that L_0 is a Lie algebra and L_1 is an L_0-module. In Lie theory one has obvious analogs of many ring theory concepts (e.g., ideals, quotients, homomorphisms) and some group theory concepts (e.g., solvability, nilpotence, simplicity). A great achievement was the classification of the finite dimensional simple Lie algebras over $\mathbb{C}$ by Cartan. Up to isomorphism, each such algebra is uniquely determined by an integral "Cartan matrix," $A = [a_{ij}]$, $1 \le i, j \le l$, where $a_{ii} = 2$, $a_{ij} \le 0$ for $i \ne j$, and $a_{ij} = 0$ iff $a_{ji} = 0$.

A basic theorem of Serre gives a description of L by generators and relations from the data in the Cartan matrix. These *finite type* Cartan matrices, all positive definite, fall into four infinite classes, A_l, B_l, C_l, D_l, and five exceptional cases, E_6, E_7, E_8, F_4, and G_2. Kac [K1] and Moody [Mo1] independently developed the idea of using any integral matrix satisfying the above conditions to define a Lie algebra with Serre's generators and relations. In the case where the matrix is degenerate but every submatrix is of finite type, called an "affine" matrix, the resulting infinite dimensional affine Lie algebra can be described explicitly as follows. Let $\mathbf{g}$ be a finite dimensional simple Lie algebra and let $\mathbb{C}[t, t^{-1}]$ be the ring of polynomials in t and t^{-1}. For $x \in \mathbf{g}$ and $n \in \mathbb{Z}$ let $x(n) = x \otimes t^n$ in the "loop algebra" $\mathbf{g} \otimes \mathbb{C}[t, t^{-1}]$. Let

$$\tilde{\mathbf{g}} = \mathbf{g} \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}c$$

be the central extension of the loop algebra with the brackets

$$[x(m), y(n)] = [x, y](m+n) + m\langle x, y\rangle \delta_{m,-n} c$$

where $\langle x, y\rangle$ is the Killing form on $\mathbf{g}$ normalized so that $\langle \alpha, \alpha\rangle = 2$ for long roots α. An affine Kac-Moody algebra is either such a $\tilde{\mathbf{g}}$ or a subalgebra obtained from it by a finite order automorphism of $\mathbf{g}$. In either the explicit description or the generators and relations definition, it is necessary to form the extension $\hat{\mathbf{g}} = \tilde{\mathbf{g}} \oplus \mathbb{C}d$ by adjoining a derivation d such that $[d, x(m)] = m\, x(m)$. Then with respect to the Cartan subalgebra spanned by c, d and a Cartan subalgebra $\mathbf{h}$ of $\mathbf{g}$, one has an affine root system Δ, an affine Weyl group W and simple roots $\alpha_0, \alpha_1, \dots, \alpha_l$, which generate Δ. In fact, it is natural to adjoin the Witt algebra spanned by all the derivations $d(n)$, $n \in \mathbb{Z}$, such that $[d(n), x(m)] = m\, x(m+n)$ and $[d(m), d(n)] = (n-m)\, d(m+n)$. On highest weight representations of $\hat{\mathbf{g}}$ one has the Sugawara construction of operators L_m, $m \in \mathbb{Z}$, representing a central extension of the Witt algebra called the Virasoro algebra. The commutator of L_n with the operator representing $x(m)$ is the operator representing $m\, x(m+n)$, but one has $[L_m, L_n] = (n-m)\, L_{m+n} + \frac{1}{12}(m^3 - m)\delta_{m,-n} c'$ for central element c'. The construction of representations of the Virasoro algebra is very important in conformal field theory and provides an essential link between that subject and the representation theory of affine Kac-Moody algebras.

Any Kac-Moody algebra has a root system, a Weyl group, simple roots, and highest weight representations. For infinite dimensional Kac-Moody algebras which are not affine one does not have at this time an explicit description of the whole algebra. It is a very important open problem to discover such a description, or even to elucidate the dimensions of the root spaces. When the Cartan matrix is "symmetrizable," that is, when there exists a rational diagonal matrix D such that DA is symmetric, then the generators and relations description suffices to prove analogs of the Weyl character and denominator formulas. The denominator formula looks like a summation over the Weyl group equals a product over the positive roots, explicitly involving the root multiplicities, and implicitly determining them. In the affine case, where all ingredients are known, this gives the famous Macdonald identities for powers of the Dedekind η-function [Mac, K2, K3, L1, L2, Mo3]. The character formula is a multi-variable power series identity whose specializations can give many combinatorial identities [FL, LMi, L3, L5]. For the simplest affine algebra $A_1^{(1)}$, which is $\hat{\mathbf{g}}$ for $\mathbf{g} = sl(2)$, the representation theory gives the Rogers-Ramanujan identities and infinitely many generalizations due to Gordon, Andrews, and Bressoud [LW2–LW6]. For rank three hyperbolic Kac-Moody algebras, it is very interesting to note that the Weyl groups are hyperbolic triangle groups [Yo] ($PGL(2, \mathbb{Z})$ in one case). The connection between characters of highest weight modules for affine algebras and modular functions has been extensively investigated [Fr3, K4, KP1, KP3, KW1, KW2, L4], and plays an important role in applications to physics. For rank two hyperbolic algebras there is a connection with Hilbert modular forms [LMo], and for the rank three hyperbolic algebra with Weyl group $W = PGL(2, \mathbb{Z})$, there is a connection with Siegal modular forms [FF1].

Any irreducible highest weight representation of a Kac-Moody algebra can be constructed as the quotient of a Verma module by its maximal proper submodule. This construction suffices for some purposes, but in some cases other constructions are known which provide much more information and give a connection with physics [FF2, Fr2]. Two such constructions are known as the "vertex" [LW1, FK, KKLW] and the "spinor" [Fr1, KP2]. Since they have such serious applications, these constructions have a reputation among the general mathematical public for being difficult. The techniques needed for proofs and the twisted versions [KP4, L6] may somewhat justify such a reputation, but the constructions are essentially simple, elegant, and easy to describe.

In its simplest form, the vertex construction gives a representation of $\hat{\mathbf{g}}$ for $\mathbf{g}$ of type A, D, or E, from the even integral root lattice Λ of $\mathbf{g}$. A key ingredient is the "bosonic Fock space" representation of the infinite dimensional Heisenberg subalgebra $\hat{\mathbf{h}} = \mathbf{h} \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}c$. For $h \in \mathbf{h}$, the "creation" ("annihilation") operators $h(n)$, $n < 0$ ($n > 0$) act by multiplication (differentiation) on the symmetric algebra $S(\hat{\mathbf{h}}^-)$ of polynomials in the commuting variables $h(n)$, $n < 0$. To get a representation of $\hat{\mathbf{g}}$ it is necessary to extend this space by tensoring with the group algebra of the root lattice, $\mathbb{C}[\Lambda]$. Let α be a root of $\mathbf{g}$ and let x_α be a root vector which is part of a Chevalley basis of $\mathbf{g}$. The vertex operators $Y_n(e^\alpha)$ representing $x_\alpha(n) \in \hat{\mathbf{g}}$ for $n \in \mathbb{Z}$ on

$V = S(\hat{\mathbf{h}}^-) \otimes \mathbb{C}[\Lambda]$ are the coefficients of the generating function

$$Y(e^\alpha, z) = \exp\left(\sum_{m\geq 1} \alpha(-m)\frac{z^m}{m}\right)\exp\left(-\sum_{m\geq 1}\alpha(m)\frac{z^{-m}}{m}\right)e^\alpha z^{\alpha(0)}\epsilon_\alpha$$
$$= \sum_{n\in\mathbb{Z}} Y_n(e^\alpha)z^{-n-1}.$$

The only details remaining to be given are how the operators e^α, $\alpha(0)$, $z^{\alpha(0)}$, and ϵ_α act on V. They all act on the group algebra $\mathbb{C}[\Lambda]$, which has basis $\{e^\lambda | \lambda \in \Lambda\}$. The operator e^α acts by multiplication, $e^\alpha \cdot e^\lambda = e^{\alpha+\lambda}$, and the operator $\alpha(0)$ acts by $\alpha(0) \cdot e^\lambda = \langle \alpha, \lambda \rangle e^\lambda$ so $z^{\alpha(0)} \cdot e^\lambda = z^{\langle \alpha, \lambda\rangle} e^\lambda$. The operator ϵ_α acts by $\epsilon_\alpha \cdot e^\lambda = \epsilon(\alpha, \lambda) e^\lambda$ where $\epsilon : \Lambda \times \Lambda \to \{\pm 1\}$ is a 2-cocycle (which can always be chosen to be bilinear) representing the unique cohomology class in $H^2(\Lambda, \{\pm 1\})$ such that $\epsilon(\alpha, \lambda)\, \epsilon(\lambda, \alpha)^{-1} = (-1)^{\langle \alpha, \lambda\rangle}$.

It is also easy to give the generating function $L(z) = \sum_{n\in\mathbb{Z}} L_n z^{-n}$ whose coefficients are operators representing the Virasoro algebra on V. For $h \in \mathbf{h}$ define the generating function $h(z) = \sum_{n\in\mathbb{Z}} h(n)\, z^{-n}$. For $h_1(m), h_2(n) \in \hat{\mathbf{h}}$ define the "bosonic normal ordering": $h_1(m)h_2(n) : = h_1(m)h_2(n)$ if $n > m$, $\frac{1}{2}(h_1(m)h_2(n) + h_2(n)h_1(m))$ if $n = m$, and $h_2(n)h_1(m)$ if $n < m$. Let $\{h_1, \ldots, h_l\}$ be an orthonormal basis of $\mathbf{h}$ with respect to the form $\langle\ ,\ \rangle$. Then we have

$$L(z) = -\tfrac{1}{2} \sum_{1\leq i\leq l} : h_i(z)h_i(z) : \ .$$

Of course, there are many important developments in representation theory that involve modifications of the simplest constructions [Ben]. In particular, one has twisted versions from automorphisms of $\mathbf{g}$ having different gradings, and one has constructions of the nonsimply-laced affine algebras [BT, GNOS], including the exceptional ones. Perhaps the most significant extensions of these ideas are the "vertex operator algebras" [Bo, FLM, FFR], their modules, and intertwining operators between modules. These are new kinds of algebraic structures, which can be constructed by making generating functions of operators $Y(v, z)$ for each vector $v \in V$, incorporating and vastly extending both the affine algebras and the Virasoro algebra. They are at the heart of conformal field theory, the Frenkel-Lepowsky-Meurman construction of the Monster Moonshine module [FLM], and the new developments in knot theory [J, TK, W, YG]. These topics are all beyond the scope of Cornwell's book.

Let me conclude with a few critical remarks about the book under review. I expected the author to have complete mastery of the mathematical aspects of the material. For the most part that expectation was satisfied, but I did find a few glaring errors, which somewhat shook my confidence in the author. On page 51 he gives an incorrect definition of differentiability of a Grassmann-valued function on an open subset of $\mathbb{R}^m$. Even allowing for an obvious misprint, his definition involves a possible division by zero. On page 238 he defines a subalgebra as the "union" of two other subalgebras instead of their sum. On page 310 he makes the statement that the parameter t in $\mathbf{g}\otimes\mathbb{C}[t, t^{-1}]$ is a real number. That strange interpretation would make the tensor product collapse to $\mathbf{g}$. Of course the book has a few minor misprints, but nothing which would seriously bother an alert reader. Despite the fact that it is written for an audience of physicists, this book can be of use to mathematicians, but I cannot give it a strong positive recommendation.

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ALEX J. FEINGOLD
STATE UNIVERSITY OF NEW YORK, BINGHAMTON