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## A GENERAL CORRESPONDENCE BETWEEN DIRICHLET FORMS AND RIGHT PROCESSES

SERGIO ALBEVERIO AND ZHI-MING MA

### 1. Introduction

The theory of Dirichlet forms as originated by Beurling-Deny and developed particularly by Fukushima and Silverstein, see e.g. [Fu3, Si], is a natural functional analytic extension of classical (and axiomatic) potential theory. Although some parts of it have abstract measure theoretic versions, see e.g. [BoH] and [ABrR], the basic general construction of a Hunt process properly associated with the form, obtained by Fukushima [Fu2] and Silverstein [Si] (see also [Fu3]), requires the form to be defined on a locally compact separable space with a Radon measure $m$ and the form to be regular (in the sense of the continuous functions of compact support being dense in the domain of the form, both in the supremum norm and in the natural norm given by the form and the $L^2(m)$-space). This setting excludes infinite dimensional situations.

In this letter we announce that there exists an extension of Fukushima-Silverstein's construction of the associated process to the case where the space is only supposed to be metrizable and the form is not required to be regular. We shall only summarize here results and techniques, for details we refer to [AM1, AM2]. Before we start describing our results let us mention that some work on associating strong Markov processes to nonregular Dirichlet forms had been done before, by finding a suitable representation of the given nonregular form as a regular Dirichlet form on a suitable compactification of the original space. In an abstract general setting this was done by Fukushima in [Fu1]. The case of local Dirichlet forms in infinite dimensional spaces, leading to associated diffusion processes, was studied originally by Albeverio and Høegh-Krohn in a rigged Hilbert space setting [AH1–AH3], under a quasi-invariance and smoothness assumption on $m$. This work was extended by Kusuoka [Ku] who worked in a Banach space setting. Albeverio and Röckner [ARÖ1–4] found a natural setting in a Souslin space, dropping the quasi-invariance assumption.

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They also derived the stochastic equation satisfied by the process (for quasi–every initial condition) (the ultimate result in this direction is contained in [ARÖ5], where also a compactification procedure by Schmuland [Sch] and a tightness result of Lyons-Röckner [LR] are used). For further results in infinite dimensional local Dirichlet forms see also [BoH, So1, So2, FaR, Tak, Fu4]. The converse program of starting from a "good" Markov process and associating to it a, nonnecessarily regular, Dirichlet form has been pursued by Dynkin [D1, D2], Fitzsimmons [Fi1], [Fi2], Fitzsimmons and Getoor [FG], Fukushima [Fu5], and Bouleau-Hirsch [BoH]. For further work on general Dirichlet forms see also Dellacherie-Meyer [DM], Kunita-Watanabe [KuW], and Knight [Kn].

Our approach differs from all the above treatments, in that we construct *directly* a *strong Markov process* starting from a given Dirichlet form without the assumption of regularity. In fact we manage to extend the construction used in [Fu3, Chapter 6], to the nonregular case. By so doing we obtain necessary and sufficient conditions for the existence of a certain right process, an $m$-perfect process as explained below, properly associated with the given, regular or nonregular, Dirichlet form. Our construction relies on a technique we have developed in [AM1] to associate a quasi-continuous kernel, in a sense explained below, to a given semigroup (this is related to previous work by Getoor [Ge1] and Dellacherie-Meyer [DM]). Our method of construction of the process is related to some work by Kaneko [Ka] who constructed Hunt processes by using kernels which are quasi-continuous with respect to a $C_{r,p}$-capacity. We also mention that our work provides an extension of a result by Y. Lejan, who obtained in [Le1, Le2] a characterization of semigroups associated with Hunt processes: in fact Lejan's work [Le1, Le2] provided an essential idea for our proof of the necessity of the condition (ii) in the main theorem below.

## 2. Main results

We shall now present briefly our main results. Let $X$ be a metrizable topological space with the $\sigma$-algebra $\mathscr{X}$ of Borel subsets. A cemetery point $\Delta \notin \mathscr{X}$ is adjoined to $\mathscr{X}$ as an isolated point of $X_\Delta \equiv X \cup \{\Delta\}$. Let $(X_t) = (\Omega, \mathscr{M}, \mathscr{M}_t, X_t, \Theta_t, P_x)$ be a strong Markov process with state space $(X_\Delta, \mathscr{X}_\Delta)$ and life time $\zeta \equiv \inf\{t \geq 0 | X_t = \Delta\}$, where, as in the usual notations of e.g. [BG], $(\Omega, \mathscr{M})$ is a measurable space, $(\mathscr{M}_t, t \in [0, \infty])$ is an increasing family of sub $\sigma$-algebras of $\mathscr{M}$, $\Theta_t$ being the shift and $P_x$ being the "start measure" (i.e. the measure for the paths conditioned to start at $x$) on $(\Omega, \mathscr{M})$, for each $x \in X_\Delta$. We denote by $(P_t)$ the transition function of $(X_t)$ and by $(R_\alpha)$, the resolvent of $(X_t)$, i.e.

$$P_t f(x) = E_x[f(X_t)]$$

and

$$R_\alpha F(x) = E_x\left[\int_0^\infty e^{-\alpha t} f(X_t)\, dt\right],$$

where $x \in X$, $E_x$ being the expectation with respect to $P_x$, for all functions $f$ for which the right-hand sides make sense.

We call $X_t$ a *perfect process* if it satisfies the following properties:

(i) $X_t$ has the normal property: $P_x(X_0 = x) = 1$, $\forall x \in X_\Delta$;

(ii) $X_t$ is right continuous: $t \to X_t(\omega)$ is a right continuous function from $[0, \infty)$ to $X_\Delta$, $P_x$ a.s., for all $x \in X_\Delta$;

(iii) $X_t$ has left limits up to $\zeta$: $\lim_{s\uparrow t} X_s(\omega) := X_{t-}(\omega)$ exists in $X$, for all $t \in (0, \zeta(\omega))$, $P_x$-a.s., $\forall x \in X$;

(iv) $X_t$ has a regular resolvent in the sense that $R_1 f(X_{t-}) I_{\{t<\zeta\}}$ is $P_x$-indistinguishable from $(R_1 f(X_t))_- I_{\{t<\zeta\}}$ for all $x \in X$ and for all $f$ in the space $b\mathscr{X}$ of all bounded $\mathscr{X}$-measurable functions. We have set $(R_1 f(X_t))_- I_{\{t<\zeta\}} := \lim_{s\uparrow t} R_1 f(X_s) I_{\{t<\zeta\}}$ (where we always make the convention that $Z_{0-} = Z_0$ for any process $Z_t$, $t \geq 0$).

*Remarks.* (i) A strong Markov process satisfying only (i), (ii) will be called a *right process* with Borel transition semigroup. If $X$ is a Radon space this definition coincides with the one in [Sh, Definition (8.1)] and [Ge2, (9.7)].

(ii) A special standard process in the sense of [Ge2, (9.10)] and, in particular, a Hunt process (cf. e.g. [BG]), always satisfy (iii), (iv), hence they are particular perfect processes (see [Ge2 (7.2), (7.3)], [Sh (4.7.6) and (9.7.10)] for the proof). Thus we have the following inclusions, cf. [Ge2, p. 55]:

$$(\text{Feller}) \subset (\text{Hunt}) \subset (\text{special standard}) \subset (\text{perfect}) \subset (\text{right}).$$

In what follows we shall assume that $m$ is a $\sigma$-finite Borel measure on $X$. A process $(X_t)$ is called *m-tight* if there exists an increasing sequence of compact sets $\{K_n\}$, $n \in \mathbf{N}$ of $X$ such that

$$P_x\{\lim_n \sigma_{X-K_n} \geq \zeta\} = 1, \qquad m\text{-a.e. } x \in X,$$

where for any subset $A$ of $X_\Delta$, $\sigma_A := \inf\{t > 0 : X_t \in A\}$ is the hitting time of $A$.

A process $(X_t)$ is called an *m-perfect process* if it is a perfect process and is $m$-tight. It follows from an idea of T. J. Lyons and M. Röckner [LR] that any strong Markov process $(X_t)$ in a metrizable Lusin space is $m$-tight if it satisfies (ii), (iii), see [AMR]. Thus in a metrizable Lusin the concepts of perfect process and $m$-perfect process coincide. (In the special case of $(X_t)$ being a standard process on a locally compact metrizable space the above conclusion that $(X_t)$ is $m$-tight can also be derived from [BG, (9.3)].)

Let us now give the correlates of above definitions for Dirichlet forms. Let $(\mathscr{E}, \mathscr{F})$ be a Dirichlet form on $L^2(X, m)$, i.e. $\mathscr{E}$ is a positive, symmetric, closed bilinear form on $L^2(X, m)$ such that unit contractions operate on $\mathscr{E}$ (i.e. $\mathscr{E}(f^\#, f^\#) \leq \mathscr{E}(f, f)$, $f^\# = (f \vee 0) \wedge 1$, all $f$ in the definition domain $\mathscr{F}$ of $\mathscr{E}$). We set as usual $\mathscr{E}_1(f, g) \equiv \mathscr{E}(f, g) + (f, g)$, $\forall f, g \in \mathscr{F}$, where $(f, g)$ is the $L^2(X, m)$-scalar product of $f$ and $g$.

In the sequel we always regard $\mathscr{F}$ as a Hilbert space equipped with the inner product $\mathscr{E}_1$. For any closed subset $F \subset X$, we set

$$\mathscr{F}_F \equiv \{f \in F : f = 0 \ m\text{-a.e. on } X - F\},$$

$\mathscr{F}_F$ is then a closed subset of $\mathscr{F}$.

The following definitions are extensions of corresponding definitions in [Fu3]. An increasing sequence of closed sets $\{F_k\}_{k\geq 1}$ is called an *$\mathscr{E}$-nest* if $\bigcup_k \mathscr{F}_{F_k}$ is $\mathscr{E}_1$-dense in $\mathscr{F}$. A subset $B \subset X$ is said to be *$\mathscr{E}$-polar* if there exists an $\mathscr{E}$-nest $\{F_k\}$ such that

$$B \subset \bigcap_k (X - F_k).$$

A function $f$ on $X$ is said to be *$\mathscr{E}$-quasi-continuous* if there exists an $\mathscr{E}$-nest $\{F_k\}$ such that $f_{|F_k}$, the restriction of $f$ to $F_k$, is continuous on $F_k$ for each $k$.

It is not difficult to show that every $\mathscr{E}$-polar set is $m$-negligible. We denote by $T_t$ resp. $G_\alpha$ the semigroup resp. resolvent on $L^2(X, m)$ associated with $(\mathscr{E}, \mathscr{F})$; i.e., if $A$ is the generator of $T_t$,

$$\left(\sqrt{-A}f, \sqrt{-A}g\right) = \mathscr{E}(f, g) \quad \forall f,\ g \in \mathscr{F} = D\left(\sqrt{-A}\right).$$

(cf. [Fu3]). We set

$$\mathscr{H} = \{h: h = G_1 f \text{ with } f \in L^2(X; m),\ 0 < f \leq 1 m\text{-a.e.}\}$$

We remark that $\mathscr{H}$ is nonempty because we assumed $m$ to be $\sigma$-finite. For $h \in \mathscr{H}$ we define the *$h$-weighted capacity* $\mathrm{Cap}_h$ as follows

$$\mathrm{Cap}_h(G) := \inf\{\mathscr{E}_1(f, f): f \in \mathscr{F},\ f \geq hm\text{-a.e. on } G\},$$

for any open subset $G$ of $X$, and

$$\mathrm{Cap}_h(B) := \inf\{\mathrm{Cap}_h(G): G \supset B,\ G \text{ open}\}$$

for any arbitrary subset $B \subset X$. It is possible to show, see [AM2], that $\mathrm{Cap}_h$ is a Choquet capacity enjoying the important property of countable subadditivity. The relation between this notion of capacity and the notion of $\mathscr{E}$-nest is expressed by the following proposition:

**Proposition.** *An increasing sequence of closed subsets $\{F_k\}$ of $X$ is an $\mathscr{E}$-nest if and only if for some $h \in \mathscr{H}$ (hence for all $h \in \mathscr{H}$) one has*

$$\mathrm{Cap}_h(X \setminus F_k) \downarrow 0 \quad \text{as } k \to \infty.$$

For the proof we refer to Proposition 2.5 of [AM2].

Denote by Cap the usual 1-capacity given by $\mathscr{E}$, cf. [Fu3]. One has obviously $\mathrm{Cap}_h(B) \leq \mathrm{Cap}(B)$ for every $B \subset X$. Hence the following corollary holds:

**Corollary.** *Every set $B \subset X$ with* Cap $B = 0$ *is an $\mathscr{E}$-polar set. Every nest $\{F_k\}$ resp. every quasi-continuous function in the sense of* [Fu3] *is an $\mathscr{E}$-nest resp. an $\mathscr{E}$-quasi-continuous function in our sense (we remark that* [Fu3] *the space $X$ is supposed to be locally compact separable and $m$ to be supported by $X$ and Radon).*

Now let $(X_t)$ be a Markov process with transition function $P_t$. We say that $(X_t)$ is *associated with* $\mathscr{E}$ if $P_t f = T_t f$ $m$-a.e. for all $f \in L^2(X, m)$, $t > 0$, and it is properly *associated with* $\mathscr{E}$ if $P_t f$ is an $\mathscr{E}$-quasi-continuous version of $T_t f$ for all $f \in L^2(X, m)$, $t > 0$. The main result we obtain is the following

**Theorem.** *Let $(\mathscr{E}, \mathscr{F})$ be a Dirichlet form on $L^2(X; m)$. Then the following family of conditions* (i)–(iii) *is necessary and sufficient for the existence of an $m$-perfect process $(X_t)$ associated with $\mathscr{E}$:*

(i) *there exists an $\mathscr{E}$-nest $\{X_k\}$ consisting of compact subsets of $X$;*
(ii) *there exists an $\mathscr{E}_1$-dense subset $\mathscr{F}_0$ of $\mathscr{F}$ consisting of $\mathscr{E}$-quasi-continuous functions;*
(iii) *there exists a countable subset $B_0$ of $\mathscr{F}_0$ and an $\mathscr{E}$-polar subset $\mathscr{N}$ such that*

$$\sigma\{u: u \in B_0\} \supset \mathscr{B}(X) \cap (X - N).$$

*Moreover, if an m-perfect process* $(X_t)$ *is associated with* $\mathscr{E}$ *, then it is always properly associated with* $\mathscr{E}$ *.*

*Remarks.* (i) If $\mathscr{E}$ is a regular Dirichlet form in the sense of [Fu3] then all conditions are satisfied. But the regularity assumption on the Dirichlet form $\mathscr{E}$ usually assumed in the literature, cf. [Fu3, Si], is not necessary for the existence of an $m$-perfect process (it is even not necessary for the existence of a diffusion process). Assume in fact each single point set of $X$ is a set of zero capacity (e.g. $X = \mathbf{R}^d$, $d \geq 2$, $\mathscr{E}$ the classical Dirichlet form associated with the Laplacian on $\mathbf{R}^d$). Let $\mu$ be a smooth measure (in the sense of [Fu3]), which is nowhere Radon (i.e. $\mu(G) = \infty$ for all nonempty open subsets $G \subset X$): the existence of such nowhere Radon smooth measures has been proven in [AM4] (see also [AM3, AM5]).

We consider the perturbed form $(\mathscr{E}^\mu, \mathscr{F}^\mu)$ defined as follows:

$$\mathscr{F}^\mu := \mathscr{F} \cap L^2(X; m); \qquad \mathscr{E}^\mu(f, g) := \mathscr{E}(f, g) + \int_X f g \mu(dx) \quad \forall f, \ g \in \mathscr{F}^\mu.$$

It has been proven in [AM5, Proposition 3.1] that $(\mathscr{E}^\mu, \mathscr{F}^\mu)$ is again a Dirichlet form. One can check that $(\mathscr{E}^\mu, \mathscr{F}^\mu)$ satisfies all conditions in the theorem, see [AM7], hence the theorem is applicable and there exists an $m$-perfect process associated with $(\mathscr{E}^\mu, \mathscr{F}^\mu)$. Moreover, if $(\mathscr{E}, \mathscr{F})$ satisfies the local property, then so does $(\mathscr{E}^\mu, \mathscr{F}^\mu)$ and hence there exists a diffusion process associated with $(\mathscr{E}^\mu, \mathscr{F}^\mu)$ (see (ii) below). But clearly $(\mathscr{E}^\mu, \mathscr{F}^\mu)$ is not regular, in fact there is even no nontrivial continuous function belonging to $\mathscr{F}^\mu$, since $\mu$ is nowhere Radon. See [AM7] for details.

(ii) An $m$-perfect process is a diffusion (i.e. $P_x\{X_t$ is continuous in $t \in [0, \zeta)\} = 1$, for q.e. $x \in X\}$) if and only if the associated Dirichlet form $(\mathscr{E}, \mathscr{F})$ satisfies the local property (in the sense that $\mathscr{E}(f, g) = 0$ if $\operatorname{supp} f \cap \operatorname{supp} g = \varnothing$), see [AM6].

(iii) By requiring $\mathscr{F}_0$ in (ii) to consist of strictly $\mathscr{E}$-quasi-continuous functions we obtain a necessary and sufficient condition for the existence of a Hunt process associated with $(\mathscr{E}, \mathscr{F})$, see [AM8].

(iv) By introducing a dual $h$-weighted capacity and employing a Ray-Knight compactification method it is possible to extend the above theorem to nonsymmetric Dirichlet forms satisfying the sector condition.

(v) Applications of the above theorem to infinite dimensional spaces $X$ are in preparation. They allow in particular to construct infinite dimensional processes with discontinuous sample paths, with applications to certain systems with infinitely many degrees of freedom.

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*Note added in Proof.* After we finished this work we learned from P. J. Fitzsimmons that every (nearly) $m$-symmetric right process is an $m$-special standard process (see [Fi1]), and our notion of perfect process is equivalent to a special standard process. Thus our theorem gives in fact a general correspondence between Dirichlet forms and right processes. We are most grateful to P. J. Fitzsimmons for his remark.

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CERFIM, Cas. Post. 741, CH 6900 Locarno, Switzerland

*Current address*: Faculty of Mathematics and SFB 237, Ruhr University, D 4630 Bochum, Federal Republic of Germany

BiBos, University of Bielefeld, D 4800 Bielefeld, Federal Republic of Germany

*Current address*: Institute of Applied Mathematics, Academia Sinica, Beijing 100080, China