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Nonlinear wave equations, by Walter A. Strauss. American Mathematical Society, Providence, RI, 1989, 91 pp. $21.00. ISBN 0-8218-0725-0

This book is 73rd in the series published by AMS and each one is based on a one week series of lectures concentrated on a part of a field. This volume deals with a very fundamental area of nonlinear equations to which the author himself has contributed very much. A linear wave equation is defined to be an equation with periodic solutions and a nonlinear wave equation, as exemplified by the nonlinear wave equation $UH - \Delta U + f(U) = 0$ and the nonlinear Schrödinger $iU_t - \Delta U + f(U) = 0$, has lower order terms added to the linear part. The investigation of such equations was first suggested by Heisenberg, with $f = U + U^3$ for the nonlinear Klein Gordon equation, as a first attempt to understand nonlinear scattering. What is surprising is the width of phenomena these equations display. Some have a scattering theory, some have blow-up, some have nonexistence. The book in hand is an excellent guide to the subject distinguishing the five points of the solution behavior. It is also a handy reference book to have on your shelf. It begins with an introduction consisting of a short instructive survey and a useful review of the key linear estimates. A chapter on invariant transformations and conservation laws follows. From these laws a great many solution properties can be derived. In Chapter 3, various existence theories are presented along with some brief but crucial proofs. Chapter 4 deals with the phenomena of "blow-up," the study of which goes back to a 1957 paper of J. B. Keller but which was placed on a firm analytic basis by the work of F. John. The author points out that most results of this kind show that the solution does not exist beyond some time but a few do demonstrate that the solution becomes singular. All this is of interest to physicists because of the connection still not quite understood to relativity. Chapter 5 presents small amplitude (this corresponds to weak nonlinearity) theory, which is often the only theory that has been completed. Chapter 6 is on scattering theory where the author presents current results and a version of our joint work for the NLKG equation.

Since many of these nonlinear wave equations can easily be seen to have solitary waves, for completeness we have Chapter 7, which looks at their stability.

Chapter 8 branches out to look at the Yang Mills equation. These can be somewhat untidy but the author presents this system in an elegant way (equation (5) p. 68 should have B for H) and describes the important properties.

The final chapter branches in another direction to the relativistic Maxwell-Vlasov system, which describes rarefied plasmas with some higher speed charged particles. This system is nonlinear hyperbolic of the semilinear kind. But it has some very special properties. Ron Di Perna, to whose memory this book is dedicated, and P. L. Lions have an interesting brief proof of the existence of a weak solution for the initial value problem that is given here. The chapter ends with companion results connected also to uniqueness.

Because of its briefness, 86 packed pages, there is little background and a limited connection to physical problems. One can only hope that Strauss will some day write a longer book that will have much more background material and display his excellent expository abilities again.

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Homological Questions in Local Algebra, by Jan R. Strooker. Cambridge University Press, Cambridge 1990, 307 pp., $34.50. ISBN 0-521-31526-3

In the late 1950s the use of homological methods revolutionized commutative noetherian ring theory. Maurice Auslander, David Buchsbaum, Jean-Pierre Serre, and others used homological methods to solve several open problems in commutative algebra. New questions were suggested by their work and, in some cases, were conjectured by them. These questions became known as the homological conjectures; other problems that grew from the original list were later added.

Perhaps the most famous problem solved during this time was the proof, due to Auslander and Buchsbaum [AB], that regular local rings have the unique factorization property. Regular local rings are the generalization of the local rings at smooth points in algebraic varieties. They are defined by the condition that the minimal number of generators of their unique maximal ideal is equal to the dimension of the ring (see Definition 1.2). It is always true that the minimal numbers of generators of the maximal ideal is at least the dimension of the ring (see e.g., Krull's theorem below).

During the 1960s some progress was made on the homological conjectures. Nonetheless, there was not a great deal of progress until the late 1960s. (Actually, many were not stated until the 1970s). With hindsight, this was because the techniques to solve them were not in place. The proofs of most of these conjectures (for arbitrary noetherian rings that contain a field) require a method called "reduction to characteristic p," and exploit the Frobenius endomorphism of a ring of characteristic p that sends an element to its pth power. To achieve this reduction an important theorem of Michael Artin is needed, called the Artin Approximation Theorem [Ar], which was not proved until the late 1960s (although special cases were studied earlier by Lang and also by Greenberg). This method of reduction essentially allows one to give proofs for rings of characteristic p, then to claim the validity of the theorems for rings that contain the rationals provided the statements of the theorems can be expressed "equationally." On the other hand, the proofs of many of the homological conjectures could have been done much earlier in characteristic p and for finitely generated algebras over fields.

All rings in this review will be commutative, noetherian, and with identity. All modules will be unital. A finitely generated free complex over R is a finite complex of finitely generated free R-modules,

$$\mathbf{F}: 0 \to F_n \to F_{n-1} \to \cdots \to F_0 \to 0. \tag{1.1}$$