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BULLETIN (New Series) OF THE
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Algebraic ideas in ergodic theory, by Klaus Schmidt. Amer. Math. Soc., Providence, RI, 1990, 94 pp., \$30.00. ISBN 0-8218-0727-7

This short book divides into two parts that deal with somewhat different topics. A general description is given in the following excerpt from the author's introduction:

> ... The first of these topics is the influence of operator algebras on dynamics. The construction of factors from group actions on measure spaces introduced by F. J. Murray and J. von Neumann in the 1930s has, in turn, influenced ergodic theory by leading to H. A. Dye's notion of orbit equivalence, G. W. Mackey's study of virtual groups, and the investigation of ergodic and topological equivalence relations by W. Krieger, J. Feldman and C. C. Moore, A. Connes, and many others. The theory of operator algebras not only motivated the study of equivalence relations (or orbit structures), but it also provided some of the key ideas

> for the development of this particular branch of ergodic theory. The first four sections of these notes are devoted to ergodic equivalence relations, their properties, and their classification, and present occasional glimpses of the operator-algebraic context from which many of the ideas and techniques arose. Ergodic theorists tend to regard ergodic equivalence relations as a subject set apart from the main body of their field; for this reason I have included a large number of examples which (I hope) show that equivalence relations provided a very natural setting for many classical constructions and classification problems. Many of these examples are drawn from the context of Markov shifts; this was partly motivated ... by the ease and naturalness with which some of the most useful invariants in coding theory can be derived and interpreted from the point of view of equivalence relations.
>
> The last three sections of these notes are dedicated to higher dimensional Markov shifts, a difficult field of research with no indication yet of a satisfactory general theory. This lack of progress is all the more remarkable when compared with the richness of the theory in one dimension; it is due to a variety of reasons, the most famous of which is that any reasonably general definition of higher dimensional Markov shifts immediately leads to the problem that it may be undecidable whether the shift space is nonempty. A second reason is that none of the techniques which have been so successful for one dimensional Markov shifts, and some of which were described in the preceding sections, appear to be applicable here. Section 5 is devoted to elementary examples of such shifts and to the surprising difficulties these examples present. However, if one makes the (very restrictive) assumption that the Markov shift carries a group structure, then many of these difficulties can be resolved, and one has the beginnings of a successful analysis which turns out to encompass the theory of expansive $\mathbb{Z}^d$-actions by automorphisms of compact groups, and which exhibits an intriguing interplay between commutative algebra and dynamics (Sections 6–7) ...

The author has made substantial contributions in the areas discussed in these notes. His treatment bears a strong personal imprint and makes lively reading.

The book is valuable and timely in several respects:

1. Although there exist good treatments of parts of what—to avoid assuming ergodicity—I will call measured equivalence relations (with countable equivalence classes), this is the only place I have seen the main results collected in a systematic way.

2. The simultaneous treatment of the measured equivalence relations and Markov shifts (otherwise known as subshifts of finite type), with invariants, constructions, and theorems from the latter serving as examples of the former, makes both subjects more interesting and easier to absorb. Of course, most of these connections were known to those workers in symbolic dynamics who dealt

with operator algebras and measured equivalence relations, but it is valuable to have them spelled out for a larger audience.

3. The study of multidimensional Markov shifts, while still at an early stage, is bound to grow in importance, partly because of links with other active areas (sample buzzwords: tiling, percolation). The examples in the last three chapters, especially in chapter 5, give a wonderfully interesting first view of the subject.

My one complaint about the book is that the system of cross-referencing is unnecessarily confusing; for example, "(1.5)" has more than one meaning.

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Topics in matrix analysis, by Roger A. Horn and Charles R. Johnson. Cambridge Univ. Press, 1991, viii + 607 pp. $59.50. ISBN 0-521-30587-X

In his comprehensive historical study [10] Kline wrote:

> Though determinants and matrices received a great deal of attention in the nineteenth century and thousands of papers were written on these subjects, they do not constitute great innovations in mathematics... Neither determinants nor matrices have influenced deeply the course of mathematics despite their utility as compact expressions and despite the suggestiveness of matrices as concrete groups for the discernment of general theorems of group theory...

Despite these sentiments, doubtlessly shared by a substantial part of the mathematical public, interesting, difficult, and important work on matrix theory continues to appear at an accelerating pace. As evidence of this, within the last year or so, Brualdi and Ryser published *Combinatorial matrix theory* [5], *Abstract linear algebra* by Curtis was posthumously published [6], and the second volume of the Horn and Johnson work (H & J), the subject of this review, made its long awaited appearance.

The book literature in matrix theory exploded in the sixties and early seventies with literally dozens of rather pedestrian efforts. This deluge occurred partly in response to NSF educational initiatives that dictated a new undergraduate curriculum in which "linear algebra" and "finite math" were to be introduced at the earliest possible moment. As might have been predicted, the logical outcome in the eighties was the inclusion of elementary matrix theory in ponderous calculus books already too heavy to lift unaided. Nonetheless, significant books on matrices and their mathematical applications have appeared irregularly over the last fifty years. Bourbaki's *Algebra* [4] commits 476 pages to linear and multilinear algebra, albeit at a predictably rarefied level. Even so, Bourbaki devotes space to some very old fashioned (and hard) matrix/determinant problems; e.g., the evaluation of the Cauchy determinant $\det(a_i + b_j)^{-1}$; the Sylvester-Franke