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LASZLO FUCHS
TULANE UNIVERSITY
E-mail address: mt0mamf@vm.tcs.tulane.edu

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André Weil: The apprenticeship of a mathematician, by André Weil (translated from French by Jennifer Gage). Birkhäuser Verlag, Basel, Berlin, 1992, 197 pp., $29.50. ISBN 3-7643-2650-6

Early in 1947 there were two remarkable mathematical events.

On the one hand, a Colloquium volume on The Foundations of Algebraic Geometry by André Weil appeared; it presented a firm basis for a subject that beforehand seemed without real foundations, and it provided Weil's proof of the Riemann Hypothesis on the Zeta function for function fields.

On the other hand, new volumes of Bourbaki's Elements were published. It was suddenly clear that this redoubtable multicephalic author was indeed going to cover all the basic parts of mathematics and that this would vitally influence the way a whole generation would view the subject.

We all heard the legend: Cartan, Chevalley, Delsarte, Dieudonné, and Weil (The Founding members) visited Montmartre to find a bearded clochard muttering in his absinthe insights about compact structures and their representations. They then sat at his feet, learned all about it, and polished it up in elegant form. My files once had a splendid photo of that clochard, Nicholas Bourbaki, white beard and all.

Now, as Weil writes in this book, "The time has come to unveil these mysteries." Here, gentle reader, you will discover the real way in which this unusual and influential collaboration came about. You will also find a sensitive and insightful presentation of the development of this remarkable mathematician; without going into technical detail, this slim, well-written, and captivating volume shows the results of early exposure to the highly charged scientific milieu of Paris.

André Weil was born in Paris on May 6, 1906. By age 5 he had learned to read. His father was a physician; his mother closely supervised his early education, finding him special tutors, getting him to skip some forms (grades), and finding him a number of truly accomplished teachers. Weil fondly recalls several of these teachers, especially one M. Collin, who taught him in the first (top) form at the famous Lycée Saint Louis—and in particular, brought him to understand that, in writing mathematics, one should never say "it is obvious that". Weil studied Latin, Greek, and Sanskrit. With friendly advice from Hadamard, he studied Jordan's "Cours d'Analyse".

After the required one year preparatory course (the "taupe"), Weil entered the famous Ecole Normale Superieur at the (very early) age of 16. In three years there he reveled in the library, heard lectures by Picard and by Lebesgue, and participated in the famous seminar of Hadamard. He read Riemann on abelian functions ("not hard—every word is loaded with meaning"). Jules Bloch taught him Sanskrit. Sylvain Lévi advised him to read the Bhagavad Gita; Weil was much affected by its beauty. This, and more, was his "undergraduate" work. A model for talented youngsters here?

After graduation at age 19, Weil traveled to Italy, where he met Enriques, Severi, Lefschetz, and Zariski. He had already studied Fermat and had acquired a taste for diophantine equations. At one point, he observes that he tried ("some 50 years too soon") to settle the now-famous Mordell conjecture. In Rome, Weil was warmly received by the mathematician Vito Volterra and went to concerts with Volterra's son Eduardo. Weil describes his time in Rome in these words: "I worked in moderation, or rather, I dreamed about mathematics as I strolled about the city." He spent much time to acquaint himself with classical and contemporary Italian art; he had prepared himself for this by reading Berenson and Venturi's multivolume history of Italian art. Did Weil have thoughts on the similarities between the arts, music, and mathematics?

Weil delighted in travel. In Germany he told Courant at Göttingen about his ideas on integral equations; subsequently, Hans Lewy asked him, "Has Courant given you a topic?" Weil was thunderstruck; it had not occurred to him that one could "be given" a topic to work on. But in conversations with Emmy Noether (her courses seemed chaotic) he learned about "modern algebra", in particular, about polynomial ideals.

In Frankfurt he met Carl Ludwig Siegel and from Max Dehn learned that "mathematics was in danger of drowning in the endless streams of publications; but this flood had its source in a small number of original ideas.... If the originators of such ideas stopped publishing them, the streams would run dry." Was this 60 years ahead of its time?

Weil's thesis (1928; at age 22) extended a calculation made by Mordell for elliptic curves to apply to curves of higher genus, thus solving a 25-year-old problem of Poincaré. After the required year in the French army, Weil spent two years in India, as a professor at the Aligarh Muslim University. This stay reinforced his earlier love for Sanskrit poetry and gave him the opportunity to travel widely throughout India. He met Gandhi, at the time when Gandhi had won popularity with his civil disobedience movement, and much admired Gandhi's ideals of nonviolence; he writes, "Once I found myself among the small group of followers who accompanied him on his walk that day."

While there Weil's research on several complex variables led to a "Cauchy integral" for very general "pseudoconvex" domains. He writes (p. 91), "Every mathematician worthy of the name has experienced... the state of lucid exaltation in which one thought succeeds another as if miraculously... this feeling may last for hours at a time, even for days. Once you have experienced it, you are eager to repeat it but unable to do it at will, unless perhaps by dogged work... ."

Weil does not here enter into mathematical detail; the whole book has only one formula—Stokes' theorem, with differential forms, on page 99. The urge to find general conditions for the validity of this formula brought Weil and

Henri Cartan, both then at Strasbourg, to take the steps that led to the existence of Bourbaki. With the sense of humor common to "Archicubes" (ancient normaliens) they enjoyed making the legend and later were suitably indignant when a firm Secretary of the AMS refused to countenance AMS membership for Nicholas Bourbaki. Then in 1949 the Encyclopedia Brittanica *Book of the Year* presented an article by Ralph Boas praising a new Bourbaki volume, while observing that "as everyone knows, Bourbaki is the pen name of a group of French mathematicians". Nicholas himself responded, in a letter to the editor, about as follows: "Thank you for your kind words about my book. However, I am sad that you deny my existence. Just last year I gave a lecture to the Association for Symbolic Logic. When the authorities in the USA refused to give me a visa, the lecture was presented by my disciple, André Weil. His colleague Mac Lane, at the University of Chicago, can, I am sure, verify my existence." The editor then wrote me. My office was next door to Weil's; André saw the editor's letter and made it clear how I should respond. I did so respond. To my astonishment, I am still on good terms with the Encyclopedia.

There was also a firm rumor that Boas did not exist.

In this book we find more about Bourbaki—how Weil's wife Evelyn chose the first name "Nicholas", how Bourbaki's existence was confirmed when Elie Cartan sponsored a paper by Nicholas B. in the Comptes Rendus, and how E. Freymann published the Bourbaki volumes so that the royalties supported the many Bourbaki congresses (Pictures).

Bourbaki's organization of mathematics is surely one of the great developments of the mid-century. Weil is appropriately proud of this, citing as examples the introduction of the general notions of "structure" and "isomorphism". Actually, the phrase "structure" was used, much in the same sense, by both Garrett Birkhoff and Oystein Ore in 1935, while my battered copy of van der Waerden's *Moderne Algebra*, part I, 1930, has a clear definition of the general notion "isomorphism"—doubtless due to E. Noether. Bourbaki did much beyond this.

Weil did not wish to serve as a soldier in the second world war; he held that it was not his dharma.... This led to adventurous complications, presented here as "The war and I: A comic opera in six acts; Prelude, Finnish Fuge, Arctic Intermezzo, Under Lock and Key, Serving the Colors, A Farewell to Arms". The description of the whole sequence is fascinating, as is the observation (p. 145): "Nothing is more conducive to abstract science than prison" (at least for A. W.).

Eventually, Weil and his family, with an invitation from the New School and with help from an American diplomat, reached the United States on May 3, 1941. The Rockefeller Foundation provided him with a modest stipend. Weil hoped to find a suitable academic position here. "Other mathematicians of my generation, to whom I did not think myself inferior, had succeeded in finding such positions." Except for help from the Guggenheim foundation, such a position did not then develop; Weil was reduced to a junior job, teaching poor calculus texts, and the like, 1942–1944 at an institution which he always called the "unmentionable place". (Here not named, but identified.)

In earlier cases, this country had indeed managed to place effectively many talented refugee mathematicians from Europe—not always in the best locations, but with later success. In Weil's case, there was clearly a failure; I racked my brains to find the reason. At that period, with the war, there may not have been many openings; I recall only one tenure-track appointment then at Harvard (a faculty instructor); moreover, the University of Chicago, when offered the chance of appointing Karl Ludwig Siegel, took no action. In A. W.'s case the reason cannot be ignorance of Weil's stature; I have personal evidence to the contrary. I had lectured on algebraic functions at Harvard, using Weil's elegant proof of the Riemann-Roch theorem. I was then a member of the AMS committee to choose hour speakers for Eastern sectional meetings. At a committee meeting, I observed that an active young French mathematician was now in this country; we should certainly ask him to speak. The chairman of the department at the "unmentionable" place, also a member of that committee, was glum and silent. But Weil was invited and did address the AMS, April 28–29, 1944, on "Modern Algebra and the Riemann Hypothesis" summarizing his astounding proof of the Riemann hypothesis for function fields. The complete presentation of this and related results required the preparation of his treatise on the "Foundations". In late 1944, he and his family left the USA for a position in São Paulo, Brazil, but not before mailing to the AMS offices the completed manuscript of this book.

To see the full setting of these and other achievements, do read this fascinating account of the development of a mathematician.

SAUNDERS MAC LANE
UNIVERSITY OF CHICAGO

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Cycles and bridges in graphs, by Heinz-Jürgen Voss. Kluwer Academic Publishers, Dordrecht, Boston, London, 1991, xii + 271 pp., $112.00. ISBN 0-7923-0899-9

It would be hard to find a graph theorist who has not written at least a paper or two on some question involving cycles. Not that this should come as a great surprise, since only very special graphs (forests) contain no cycles; the fact that a graph contains cycles leads naturally to many specific questions. What is the shortest cycle? What is the longest cycle? Is there a cycle containing all of the vertices? In what ways do various graph parameters, for example, the minimum degree, influence the existence of cycles of specified length? What conditions ensure cycles with many diagonals? Graph theory has developed an array of cycle-related properties (girth, circumference, hamiltonian graph, etc.) and presents the researcher with the perpetual challenge of relating these properties to such graphical features as minimum degree, neighborhood unions, forbidden subgraphs, connectivity, planarity, etc.

Proof techniques for problems involving cycles vary in sophistication. Early results of Dirac and Ore have inspired many similar approaches. These arguments often involve high levels of creativity and technical skill but may leave