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Matching of asymptotic expansions of solutions of boundary value problems, by A. M. Ilin. Translations of Mathematical Monographs, vol. 102, American Mathematical Society, Providence, RI, 1992, ix+279 pp., $169.00. ISBN 0-8218-4561-6

This book presents a large body of work on asymptotic expansions (singular perturbation problems) of solutions to boundary value problems for ordinary and partial differential equations. The book represents the results over a number of years of a group of Russian mathematicians from the cities of Ufa and Sverdlovsk.

The methodology of the book is to present in each chapter some plausible general statements about the type of problem being considered and then illustrate all details for simple concrete examples. An unusual feature of the book is that the general term is derived for all the asymptotic expansions. Further, proofs are given that the formal expansions are truly asymptotic to the exact solution in all cases. Naturally, the problems considered in the book are linear with several exceptions noted below. Asymptotic matching is carried out systematically with a version of Van Dyke's matching procedure (not referenced) that the (m term inner limit of the n term outer limit) = (n term outer limit of the m term inner limit). This matching is exhibited in a table.

Various chapters discuss (i) singular perturbation for ordinary differential equations (ε on the highest derivative), called bisingular here if the outer expansion has singularities near the boundary. Corner layers are considered. One nonlinear first-order differential equation which has an intermediate layer, is also considered here. (ii) Singular perturbations of the domain for elliptic boundary value problems. Here the typical problem is the occurrence of a small cavity which approaches zero in the interior. (iii) Elliptic equations with a small parameter in the highest derivative, where the subcharacteristics are important. Cases of subcharacteristics tangent to the boundary in various ways are considered. (iv) Hyperbolic systems (weakly quasi-linear) where there is trouble for large times. (v) The Cauchy problem for the standard quasi-linear first-order equation, which develops shocks and the effect of a small parameter on the diffusion term; something about the ordinary differential equations of flame fronts with high-activation energy asymptotics.

The book has a series of interesting examples, a few of which produce results that might not be expected. Near the beginning of the book the author offers the opinion that there is no systematic way to find the structure of the gauge functions for asymptotic expansions, but then he proceeds to find them anyway. The notion of limits and distinguished limits does not appear.

The translation from Russian to English is generally good, although in several places, where the meaning was clear, "operator" was translated as "equation".

Only a few physical problems are discussed and then the description is very brief. This book will be of the most interest to people working in singular perturbation theory for ordinary and partial differential equations. It is a monograph with basically no references in the text, but it does contain a good bibliography, especially of Russian references with some annotation. There are no exercises or problems.

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Complexity theory of real functions, by Ker-I Ko. Birkhäuser, Basel, 1991, viii+307 pp., $49.50. ISBN 0-8176-3586-6

A striking aspect of twentieth century mathematics is the emergence of new fields of specialization motivated by the computer. The book under review may be considered in this light. It deals with a specialized topic in theoretical computer science and is written by a computer scientist. However, the mathematical roots of the subject extend back to the pioneering work on computability of the 1930s. Research in this area is still going on today.

We begin with a brief historical account. In the 1930s mathematicians became interested in studying computability in its widest sense. There were many reasons for this, some of which were implicit in the work of Kurt Gödel. For example, mathematicians wanted to understand the notion of a *mathematical proof*. Now a mathematical proof, if written down in detail, is *mechanically checkable*. Thus a definition of the most general type of mechanically checkable procedure was desirable. This led to a study of computability.

In 1936 Alan Turing defined the Turing machine, a digital computer of a very general type [6]. This gave rise to a definition of computability for functions mapping the nonnegative integers N into N. Such a function is computable if its values can be calculated by one of the machines described in [6]. Turing went further and proved the existence of a universal Turing machine—a digital computer which can simulate all others. It is worth noting that the digital computers of today are Turing machines with limited storage capacity. So it is not surprising that a study of computability based on the Turing machine is of interest to computer scientists.

At about the same time, other definitions—by Post, Herbrand/Gödel, Church, etc.—appeared. Several of these were useful in solving problems in computer science. (For example, the Post production, a rule for manipulating strings of symbols, played a role in the theory of programming languages, the work of Noam Chomsky on the grammatical analysis of language, and other topics.) A seminal result was that all of the definitions are equivalent: the functions so defined are called recursive functions. The equivalence provided evidence that the definition of computability which emerged is a good one. (There is additional evidence, which we will not discuss here.) As a consequence of the work on computability, mathematicians obtained a definition of the most general type of proof procedure, and the theory of recursive functions became the theoretical