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Measures of noncompactness and condensing operators, by R. R. Akhmerov, M. I. Kamenskii, A. S. Potapov, A. E. Rodkina, and B. N. Sadovskii. Birkhäuser Verlag, Basel, Boston, and Berlin, 1992, 249 pp., $99.50. ISBN 3-7643-2716-2

The first measure of noncompactness was introduced by K. Kuratowski [K] in 1930 in generalizing Cantor's intersection theorem. Kuratowski's measure, $\alpha(M)$, defined for bounded subsets M of a metric space, is the infimum of all numbers $\delta > 0$ such that M is a union of finite number of subsets each having diameter less than δ. He then proved the following theorem: *If* $\{F_n\}$ *is a decreasing sequence of closed, nonempty subsets of a complete metric space such that* $\lim_{n\to\infty} \alpha(F_n) = 0$, *then* $\bigcap_{n=1}^{\infty} F_n$ *is compact and nonempty.* This simple idea may be viewed as the starting point for a theory that has led to over five hundred papers in the course of the last three decades.

Kuratowski's idea apparently lay fallow for the next twenty-five years until Darbo [D] discovered that it could be used to develop a theory for a class of operators, called condensing operators, encompassing both the completely continuous and the contraction operators. Almost simultaneously with Darbo, L. S. Gol'dsteĭn, I. Gohberg, and A. S. Markus [GGM] developed a measure

similar to Kuratowski's in order to study properties of linear operators. Generally, a condensing operator has the property that images of bounded sets are more nearly compact than their preimages. For example, if $f: E \to E$ is continuous and if ψ is a real-valued measure of noncompactness for E, then f is said to be ψ-condensing with constant k provided $\psi(f(M)) \leq k\psi(M)$ for each bounded subset M of E. (The book under review requires $k < 1$.)

In the 1960s these ideas took root and flourished. Many results for compact operators were extended to condensing operators in a quite general setting. For instance, there are a Fredholm theory, index theory, and fixed-point principles both for condensing functions and for multiple-valued functions.

Many of these results for condensing operators were inspired by or have applications to infinite-dimensional problems for differential and integral equations. As an illustration of this application, consider the Cauchy problem $x' = f(t, x)$, $x(0) = 0$, on a Banach space E. For simplicity assume f is continuous and bounded on $[0, 1] \times E$. When E is finite dimensional, this problem always has at least one solution; but when E is infinite dimensional, there may exist no solutions, not even local solutions. In 1967 Ambrosetti [A] proved the existence of a solution assuming that f is uniformly continuous and satisfies $\alpha(f(t, M)) \leq k\alpha(M)$ for all $t \in [0, 1]$ and all bounded subsets M of E. A similar result was established by one of the authors (Sadovskii) at about the same time for more general measures. Since then several variations of these results have been given. A typical proof consists of showing that an associated integral operator is condensing with respect to an appropriate measure of noncompactness and then applying a fixed-point theorem for condensing operators. This approach is, of course, familiar to all of us who have used fixed-point theorems to establish properties of solutions of differential equations.

Let us now turn to the book being reviewed. This is the second book on measures of noncompactness and condensing operators. The first was a short monograph by Banaś and Goebel [BG] that appeared in 1980. The current book is a translation of a 1986 Russian book that was revised by the authors. As often happens with a translation, the book is not up to date. Of the 183 items in the references, only 21 are dated in the 1980s, with most of those dated 1980–83. The exposition largely follows the papers of the authors, who are leading contributors to this area.

The book consists of four chapters. The first gives the basic definitions of measures of noncompactness and condensing operators for Banach spaces. Chapter 2 treats the linear theory and, in particular, gives a spectral characterization of a linear condensing operator as a sum of a finite rank operator and an operator with spectral radius less than one. Perturbation results are also established. Chapter 3 is devoted to developing an index theory analogous to that for compact operators. Chapter 4, the longest chapter, considers applications to differential equations on Banach spaces, stochastic differential equations with delays, functional-differential equations of neutral type, and Hammerstein integral operators. Each of the first three chapters ends with a survey of the literature. These surveys are especially valuable in that they furnish some historical perspective and discuss extensions of the basic material presented in the chapters. In the fourth chapter notes on the references are furnished at the end of each section.

The writing is terse and in the definition-theorem-proof-remark style. Some

of the chapters are easier to read than others, and the uninitiated reader may be overwhelmed by the multitude of possible measures. Some of these measures are easier or more natural to use in certain situations than others, but not enough light is shed on what to do in a given situation. There appear to be relatively few errors. In my opinion, anyone desiring a more leisurely introduction to the area should read [BG] first.

In spite of the reservations expressed above, this book is a valuable contribution to an area that has yet to become fully focused and standardized. This effort represents a significant step towards reaching some common understanding.

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Schur's algorithm and several applications, by M. Bakonyi and T. Constantinescu. Pitman Research Notes in Mathematics Series, volume 261, Longman Scientific and Technical, Harlow, 1992, 190 pp., $54.95. ISBN 0-582-90120-9 Copublished in the U. S. by John Wiley & Sons. ISBN 0-470-21974-2

Operator methods in interpolation theory have deep classical roots which retain great importance for the modern subject. The sources for the book by Bakonyi and Constantinescu are (1) Schur's study [33] of power series which represent analytic functions which are bounded by one in the unit disk and (2) Szegö's theory [34] of orthogonal polynomials on the unit circle. The subject has evolved through a series of generalizations which are formulated in the language of Hilbert space operators, and today it is an active area with important engineering applications. All of these elements, classical and modern, are represented in the book.

Let S be the *Schur class* of analytic functions which are bounded by one on the unit disk D of the complex plane. Given $f(z)$ in S, define a sequence