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The classification of knots and 3-dimensional spaces, by Geoffrey Hemion. Oxford University Press, New York, 1992, 162 pp., $43.50. ISBN 0-19-859697-9

Faced with a collection of mathematical objects, mathematicians seem to suffer a compulsion to put them in order (with the notable exception of the preprints scattered over their desks). Hence some form of "classification problem" arises over and over in different fields of mathematics. The usefulness of a solution to a particular case of the classification problem depends on what precisely is meant by a classification and on how closely the classification is linked to the structure of the objects being classified. For example, closed orientable surfaces are completely classified by their genus, a fundamental structural property of the surface. This is a useful classification. By contrast, there is a simple algorithm to generate a list of all the prime numbers; here the list itself is not terribly informative. Nevertheless, there are certainly cases where we would be grateful simply to know that such a list existed. An algorithm to generate a complete list (without duplication) of all closed 3-manifolds, for example, would be a fine thing. An algorithm that could actually be implemented would be even more wonderful.

In this orderly spirit, knot theorists have been compiling lists of knots for decades. Almost every book on knot theory has a table of knots as an appendix [BZ, K, R]. These tables typically list all distinct knots that can be drawn in the plane with ten or fewer crossings. A triumph of new invariants [J] in knot theory, brilliant computer programming [We], and some handy work with a piece of string [P] allow us to distinguish not only among the knots in these tables but also among knots of considerably greater complexity. Nevertheless, hand a knot theorist two drawings of knots with three hundred or so crossings, and chances are excellent that he or she will be unable to decide whether the two knots are the "same"; i.e., if each were tied in a piece of string, whether one could be deformed into the other. Until the work of Haken [H] in the late 1970s there was no way, even theoretically, to make the decision. Haken's work, with a piece contributed by Hemion [He], gives an algorithm to decide whether two knots are the same and, hence, allows us to compile a complete nonduplicating knot table, i.e., to "classify" all knots.

What is Haken's algorithm? Haken proposed to classify 3-manifolds containing an "essential" surface (now called Haken manifolds) by cutting the manifold along such a surface and continuing the process, in some efficient way, until one arrived at a collection of 3-balls. He concluded that one could determine whether two Haken manifolds were homeomorphic by comparing these resulting balls together with the remnants of the surfaces along which the manifolds were cut. It is necessary not only to examine the pieces at the end of the process but also to understand how the pieces glue back together. Hemion's contribution to the problem was to devise an algorithm to classify homeomorphisms of compact orientable 2-manifolds, i.e., the surfaces along which one has to glue to reassemble the original manifold. This completed Haken's program. So Haken's algorithm, with Hemion's contribution, decides whether two Haken manifolds are homeomorphic. According to Hemion, Haken's algorithm is highly impractical (p. 4).

The book describes the application of Haken's algorithm to knot complements (plus information on the boundary) in the 3-sphere, each of which contains an essential surface (a Seifert surface). This is sufficient to produce a knot table, as follows: take any listing at all which includes all knots. For example, take the list of all drawings of knots in the plane in order of increasing crossing number. This list will have many duplicates (it would, for example, have four drawings of the unknot with a single crossing). Since any knot can be drawn in the plane with some number of crossings, the list will certainly contain all knots. To create the desired list, start at the beginning of this inefficient list and, before adding each entry, use Haken's algorithm to check whether it has already appeared. If it has, cross it out; if it has not, add it on. Notice that using this specific example, one can see that Haken's algorithm can be used not only to classify all knots but also to determine the crossing number (the smallest number of crossings necessary to draw the knot in the plane) of any knot. Despite this attractive little application, this method for producing a listing of knots has more of the flavor of the list of all primes than the list of all closed orientable surfaces.

More generally, the book is useful simply as an exposition of Haken's program for 3-manifolds containing an essential surface, i.e., an exposition of the solution to the homeomorphism problem for Haken 3-manifolds. As such, some parts of the book perform a public service. For example, in Chapters 7–9, the author gives an unusually clear explanation of normal surface theory and the algorithmic process by which one finds the essential surfaces along which to cut the manifold. Other sections of the book are less helpful and less reliable. For example, on page 13 the author decides to begin the algorithm keeping track of a longitude of the knot, when in fact he must keep track of the meridian, an odd mistake for a knot theorist. On page 45 he implies that an infinite group must map nontrivially to Z. What is most misleading, however, is the author's estimation of the importance of the classification problem for knots. He seems to regard the solution to the classification problem as having more or less finished off the field of knot theory; he compares knot theory to chess, saying "... the problem of winning a chess game can be reduced to a purely mechanical affair.... Despite this, many people continue to devote themselves to playing these finite games.... Thus in the same way that the game of chess remains interesting in practice, so is the theory of knots interesting, even though it admits a definite and finite classification procedure... " (p. 120). Without denigrating the game of chess, it is foolish to compare the deep field of knot theory, including both the recent (meaning in the last fifteen years) important work in the field (cf. Thurston, Jones, Gordon-Luecke, Gabai) and its major unsolved problems (i.e., property P) to chess. The implication is that if we simply had more powerful computers, capable of implementing this unwielding algorithm, we could abandon the field.

In short, the book is a reasonably good explanation of a particular—and important—theorem in knot theory. It gives virtually no indication of any developments in knot theory since the solution of the classification problem (1979). With the exception of the section on normal surface theory, experts in the subject would do as well to read an expository article on Haken 3-manifolds by Waldhausen [W] and Hemion's original article [He], which appears as an appendix in the book. Nonexperts should refer to books that give a better idea of the scope of the field [A, R, BZ].

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Cohomological methods in transformation groups, by Christopher Allday and Volker Puppe. Cambridge Studies in Advanced Mathematics, vol. 32, Cambridge University Press, London, 1993, xi+470 pp., $69.95. ISBN 0-521-35022-0

One of the most fundamental structures in mathematics is that of symmetry. Understanding the role played by compact Lie groups as transformation groups is a central problem in mathematics.