chapter provides an introduction to the subject, while background on homotopy theory and complex bordism can be located in the appendices as well as in the author's previous book [8]. A further appendix covers results due to Jeff Smith on the representations of the symmetric group which are needed for the proof of the periodicity theorem. The rest of the book patiently develops the subject, culminating in the proofs of the three forms of the nilpotence theorem and the periodicity theorem as well as a number of further results. The author has done a marvelous job of making difficult material accessible and inviting.

The book does contain a number of misprints and other slips. The proofs of Theorem 3.4.2 on pages 35–36 and of Corollary 5.1.5 on pages 50–51 need some reworking. Page entries in the index need to be reduced by two. Readers may wish to obtain a helpful errata listing by contacting the author at `drav@troi.cc.rochester.edu`.

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Mathematical elasticity. Vol. 1. *Three-dimensional elasticity*, by Phillippe Ciarlet. Studies in Mathematics and Its Applications, vol. 20, Elsevier Science Publishers, Amsterdam, 1988, 451 pp., $107.25. ISBN 0-444-70259-8

The mathematical foundations of elasticity theory were established in large part during the nineteenth century by mathematicians such as Euler, Cauchy, Navier, G. Green, Kelvin, and Duhem. Further distinguished contributions followed from the French and Italian schools, resulting in a comparatively sound knowledge of the basic nonlinear theory. However, for much of the first half of the present century, nonlinear aspects became increasingly neglected in favour of the linear theory. The change in emphasis was probably due to the unavailability of appropriate theorems in nonlinear analysis and differential geometry and to contemporary technology being content with predictions based on linear theory. This period, which continued until approximately the Second World War, witnessed some notable achievements recorded, for instance, in the classic books by Love on the general linear theory and by Muskhelisvili on the complex variable treatment of plane linear elasticity. The period also produced exact solutions to a wide range of special problems of importance in practical applications and, towards the end, a diversity of approximating techniques for the estimation of solutions.

Yet it had long been realised that certain experimental effects could not be satisfactorily explained solely by the linear theory, and this, combined with the postwar expanding demand for more precise results, stimulated a revival of interest in the nonlinear theory. At first, attempts were fragmentary, sometimes dependent upon heuristic assumptions of doubtful validity and employing adaptations of linear arguments. Then in the 1950s there emerged the invigorating, pioneering contributions of mathematicians such as A. E. Green, R. S. Rivlin, and C. Truesdell, who sought to structure the development of elasticity, and indeed of continuum mechanics in general, within a clear rational framework. Here were young blades in lush meadows ripe for harvest. Guided by clear physical insight, techniques from algebra, analysis, and tensorial and differential geometry were applied to the exact mathematical development of the axiomatic foundation formulated by Noll and others. The notions of strain and of stress and the constitutive relations between these quantities along with the basic invariance and balance laws were clarified and illuminated, providing an incisive understanding of these fundamental concepts and principles. Definitive versions of results were sometimes not easily agreed upon, and the ensuing discussion, often spiced by healthy controversy, helped to identify and refine essential elements in an argument. Many new theories of continuum mechanics were generated, including those of simple materials, directors, and multipolar and generalised mechanics.

The influence of these ideas has been immense, and they have attracted many distinguished scientists and engineers from North America, Europe, Japan, and elsewhere. However, by the mid-1970s there were indications that the subject was beginning to subside under the accumulation of a vast, often baroque, mathematical apparatus which many increasingly regarded as arid and remote. Physical motivation was becoming obscured, and often continuum mechanics was apparently introduced merely to justify the applicability of arcane mathematical theorems. Inevitably the initial intoxication evaporated, and progress, including that on the foundations of continuum mechanics, slowed considerably. Even today, for example, general agreement has not yet been conclusively reached on the complete formulation of plasticity and thermodynamics.

Of the new theories proposed, probably only those of liquid crystals and of non-Newtonian fluids have sustained an interest comparable to that of the classical theories of the Navier-Stokes fluid and of elasticity. These latter theories, despite any disenchantment with overelaborate use of mathematics, still remain a fertile source of intriguing and deep mathematical challenge motivated by practical questions. In particular, modern studies in elasticity are regaining the former vigour of the postwar era with genuine advances occurring not so much in the basic structure, which is almost complete, but in qualitative properties of solutions to initial and boundary value problems. But here a peculiarity presents itself. It is remarkable that activity should be sustained in a subject that at first sight is in many respects concerned with an ideal model of a physical process and thus of dubious applicability. For example, the absence of any dissipation or damping in the model means that a major observable is not taken into account; and although such effects become irrelevant in elastostatics, an equilibrium position is, after all, accessible only through a necessarily damped motion. Thus, it is questionable, for example, whether elasticity is an appropriate theory for either buckling or asymptotic (dynamic) stability. Even apart from such shortcomings, there are other constraints, similarly imposed by physics, which cannot be ignored and whose inclusion in the model render many otherwise useful mathematical theorems inapplicable or in need of severe modification. One such notable constraint is the postulate of the noninterpenetration of matter, which translates into the mathematical requirement of a positive determinant of the deformation gradient tensor. Another well-known difficulty involves the form of the strain, or stored, energy function and consequently that of the constitutive relations. Unlike the Navier-Stokes fluid where these relationships can be precisely defined, in elasticity the form of the strain energy function is still largely undetermined, being so far limited only by invariance and other requirements like growth assumptions. Indeed, a fundamental open problem is to discover restrictions on the strain energy and constitutive relations which are physically realistic and mathematically tractable. Of course, particular algebraic forms have been proposed and matched to physical data, and they provide some indication of these limitations. Ultimately, however, it is reasonable to expect that a set of necessary and sufficient conditions to be satisfied by the strain energy should follow from the assumption that the initial and boundary value problems are well posed in the sense of Hadamard. Immediately, further difficulties then arise in the implementation of this customary requirement. Firstly, global uniqueness of the solution to boundary value problems in nonlinear elasticity is undesirable, as can easily be demonstrated by the simple examples of buckling, the hemispherical annulus, and a doubly connected region. Thus, the mathematically attractive property of convexity with respect to the deformation gradient is excluded for the strain energy. On the other hand, local existence and uniqueness are to be expected, lend justification to the linearised theories, and have been established by means of the inverse and implicit function theorems. Yet again uniqueness for the initial displacement problem in the whole space and in the class of certain weak solutions has been established under the hypothesis of rank-one convexity on the strain energy. But generally the understanding of uniqueness is incomplete, and indeed classification of initial and boundary value problems where uniqueness is physically plausible and mathematically provable remains open.

Several proofs of existence in the full nonlinear boundary value problem are based upon the monotonicity of the stress or, equivalently, the convexity of the strain energy function, which, as just remarked, is an unsatisfactory assumption, since it not only contradicts nonuniqueness but also conflicts with fundamental invariance properties. A more successful approach to existence, however, is the classic device introduced by Ball [1], who, appealing to the variational calculus, required the strain energy to be poly-convex, a condition weaker than, but implying, the condition of quasi-convexity introduced by Morrey. This enables the previous difficulties to be circumvented and existence to be established without global uniqueness for the displacement, traction, and mixed boundary value problems. Existence is proved, however, only for certain weak solutions; and precise regularity of minimisers is still under active investigation, the study extending to include the practically important problems of cavitation and crack initiation. Very recently, Sverak [5] refined the relationship between poly-convexity and rank-one convexity and showed by means of an ingenious counterexample that the latter does not imply the former condition even for a quadratic strain energy density function. The converse implication has been known for some time.

The understanding of existence in the corresponding dynamic of problems is fragmentary but nonetheless crucial if, for instance, knowledge of stability is not to remain merely formal. Moreover, it is now clear that classical solutions in nonlinear elastodynamics cannot exist for all time: shock waves and other singularities develop, and hence analysis of existence must be undertaken in the class of generalised solutions. But generalised solutions may be nonunique and hence lead automatically to instability unless a selection procedure, such as the viscosity method, is used to recover uniqueness. Moreover, it is an open question as to whether the class of generalised solutions required for existence will be the same as for the class of solutions appropriate for the determination of stability, since different hypotheses may be necessary. It is likewise uncertain whether the postulates for continuous dependence upon the data in the equilibrium boundary value problems will be consistent with those of poly-convexity and others on which proofs of existence in the static problem rely.

And cutting across these traditional problems are others arising from the progressive realisation of the practical significance in nonlinear elasticity of ill-posed and inverse problems and of problems in optimisation and control. Furthermore, the number of exact solutions to both static and dynamic problems is rather meagre, providing few benchmarks for the increasing availability of computer packages.

Despite these limitations, the unresolved problems of nonlinear elasticity have retained their fascinating challenge, not only because they usually require a formidable mastery of sophisticated, advanced mathematical techniques, facilitated by judicious application of computer-aided analysis but also because nonlinear elasticity possesses ramifications for several other cognate theories of continuum mechanics. Thanks to the perception of Ericksen, there is a link with the theory of phase transitions in solids and also with the related theory of materials exhibiting memory developed by Müller which is used equally, for example, in space technology and dental science. These new areas are the focus of significant contemporary activity which, because the governing nonlinear partial differential equations may be of indefinite type, has required extensive associated developments in the variational calculus and the theory of differential equations, including the realisation of the importance of Young measures. Elasticity has also provided the motivation for extending the theory of composites which via the notion of homogenisation has likewise stimulated research in nonlinear partial differential equations. Another challenge has been the accurate derivation of the theory for lower-dimensional bodies (rods, plates, and shells) from the full three-dimensional, nonlinear theory by means of reliable, precise approximation techniques that properly consider edge effects and the related classical issue of Saint-Venant's principle and problem. A fascinating aspect of these studies is the exploitation by Miekle [3] of the "dynamical" structure of the nonlinear equilibrium equations, first remarked by Ericksen, using centre manifold and related techniques from dynamical systems.

These applications confirm both the mathematical and physical importance of elasticity as a nonlinear theory in its own right. There is a further sense, however, in which elasticity can be justified. Within solid mechanics, it occupies an archetypal, occasionally surrogate, role for several other more complex theories such as thermoelasticity, viscoelasticity, electromechanical, multipolar, and director theories. Unless problems can be resolved in elasticity, their solution is unlikely in these other contexts, and thus major attention is rightly devoted first to developments in the elastic theory.

The profound nature of the mathematical challenge presented by elasticity has resulted in steady, gradual, and sometimes spectacular advances. As already mentioned, the requisite mathematics is frequently unavailable and must be separately developed; but despite these obstacles, achievements have steadily accumulated and have been recorded in several books, commencing with the monumental monographs by Truesdell and Toupin [7] and by Truesdell and Noll [6]. At the risk of being invidious, we also mention the recent volumes by Marsden and Hughes [2] and by Ogden [4], which respectively emphasise the differential geometry and algebraic aspects of the subject. There has not been, however, any corresponding authoritative text based primarily on an analytical treatment. This omission is now remedied by the excellent book under review.

The author himself has employed analytical techniques to contribute significantly to a fundamental understanding of the subject; thus he is admirably qualified to write an extended account which, as the preface explains, is "...a thorough introduction to contemporary research in [nonlinear] elasticity, and a working textbook at the graduate level for courses in pure or applied mathematics or in continuum mechanics." Although published over six years ago, this highly commendable and readable book retains a remarkable topicality and succinctly covers most important recent developments in a self-contained manner, requiring "only basic topics from analysis and functional analysis." The present volume is the first of two, the second of which, still to be published, is planned to include a treatment of the exact derivation of the theory of lower-dimensional bodies from the full three-dimensional nonlinear theory along with comprehensive discussions of plane elasticity and nonlinear elastodynamics.

The first volume establishes the fundamentals of the subject in its opening chapters, which also deal fully with the necessary mathematical preliminaries from analysis, algebra, and geometry as well as reviewing the axioms and basic laws of continuum mechanics. These chapters also derive the main results for the three-dimensional, static theory and discuss hyperelasticity for which a strain energy function is assumed to exist.

Because of the comparative paucity of results, relatively little space is devoted to uniqueness and continuous dependence, but the rest of the subject is amply treated, with separate chapters devoted to constitutive relations, the properties and form of the strain energy function in hyperelasticity, and the formulation of boundary value problems. The latter chapter also summarises interesting results concerning both unilateral and geometrical constraints and the problems associated with self-contact and the noninterpretation of matter. Attention is properly focused on the question of existence of a solution to the standard equilibrium boundary value problems; and the two main approaches, respectively, involving the implicit function theorem and the variational calculus (minimisation techniques), are presented in clear detail within the context of functional analysis. These admirably coherent descriptions are immensely valuable to those wishing to gain a lucid introduction to these vitally important topics, whose understanding is essential, for example, to the proper design of computer packages. Somewhat inevitably, the very recent work on the phase transitions and on material defects and instabilities is omitted; but this remark is not intended as any sort of criticism, since in view of the rapid developments still occurring in these research topics, it is arguable whether they are at a stage suitable for inclusion in a book of the present kind.

Even without these latest advances, the vitality and challenge of elasticity are splendidly conveyed by the author. There is frequent mention of the major open problems, and interspersed throughout the text is a succession of remarks that illuminate and amplify the core material thoroughly developed in the form of theorems and accompanying proofs. Naturally, a first reading should be confined to the major theorems, but the experienced researcher will discover much of interest in these astute and perceptive observations. Most chapters conclude with a set of exercises complementary to the text or successfully provoking new ideas, while notation, in continuum mechanics often a formidable redoubt to comprehension, here provides considerable assistance.

The author states that his aims are to convince "the application-minded readers that analysis is indispensable for a genuine understanding of elasticity, ... especially in view of the increasing emphasis on nonlinearities" and, on the other hand, to convince "the more mathematically oriented readers that elasticity, far from being a dusty classical field, is on the contrary a prodigious source of challenging open problems." These aims are abundantly fulfilled, and the enriching interlacing of mathematics with sound physical insight firmly places the book in the tradition of those who in the postwar period set about the task of revitalising and rigorously restructuring continuum mechanics. The book, a masterly account of the subject, deserves the widest readership. The second volume is awaited with equal enthusiasm.

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The classification of knots and 3-dimensional spaces, by Geoffrey Hemion. Oxford University Press, New York, 1992, 162 pp., $43.50. ISBN 0-19-859697-9

Faced with a collection of mathematical objects, mathematicians seem to suffer a compulsion to put them in order (with the notable exception of the preprints scattered over their desks). Hence some form of "classification problem" arises over and over in different fields of mathematics. The usefulness of a solution to a particular case of the classification problem depends on what precisely is meant by a classification and on how closely the classification is linked to the structure of the objects being classified. For example, closed orientable surfaces are completely classified by their genus, a fundamental structural property of the surface. This is a useful classification. By contrast, there is a simple algorithm to generate a list of all the prime numbers; here the list itself is not terribly informative. Nevertheless, there are certainly cases where we would be grateful simply to know that such a list existed. An algorithm to generate a complete list (without duplication) of all closed 3-manifolds, for example, would be a fine thing. An algorithm that could actually be implemented would be even more wonderful.

In this orderly spirit, knot theorists have been compiling lists of knots for decades. Almost every book on knot theory has a table of knots as an appendix [BZ, K, R]. These tables typically list all distinct knots that can be drawn in the plane with ten or fewer crossings. A triumph of new invariants [J] in knot theory, brilliant computer programming [We], and some handy work with a piece of string [P] allow us to distinguish not only among the knots in these tables but also among knots of considerably greater complexity. Nevertheless, hand a knot theorist two drawings of knots with three hundred or so crossings, and chances are excellent that he or she will be unable to decide whether the two knots are the "same"; i.e., if each were tied in a piece of string, whether one could be deformed into the other. Until the work of Haken [H] in the late 1970s there was no way, even theoretically, to make the decision. Haken's work, with a piece contributed by Hemion [He], gives an algorithm to decide whether two knots are the same and, hence, allows us to compile a complete nonduplicating knot table, i.e., to "classify" all knots.