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Oriented matroids, by A. Björner, M. Las Vergnas, B. Sturnfels, N. White, and G. Ziegler. Cambridge University Press, London and New York, 1993, xii + 516 pp., $89.95. ISBN 0-521-41836-4

In 1935 H. Whitney introduced the concept of a matroid, which unifies many configurations studied in pure and applied mathematics—in particular, in algebra, geometry, combinatorics, and optimization theory. Ordinary matroids may be viewed as an abstraction of finite geometric configurations which are embedded into some vector space over a field. In 1978 oriented matroids were introduced by R. G. Bland, M. Las Vergans, J. Folkman, and J. Lawrence as combinatorial abstractions of finite geometric configurations in vector spaces over some ordered field. Since oriented matroids, as well as ordinary matroids, are important in many areas—as in algebraic and computational geometry combinatorics, topology, operations research, and chemistry—researchers in various fields were led to questions concerning oriented matroids. The purpose of the present book is to summarize the theory of oriented matroids developed thus far. Technically, the book is organized as follows:

Chapters I and II serve to motivate the definition of oriented matroids by means of connections to several branches of mathematics and natural sciences. The diverse mathematical theories all lead to cryptomorphic axiom systems for oriented matroids; the equivalence of these definitions is proved in Chapter III. It should be remarked that the proofs are not simple.

Chapters IV and V are devoted to topological representability of oriented matroids. The main results of these two chapters is the topological representation theorem which is already proved in the basic paper by J. Folkman and J. Lawrence concerning oriented matroids. It states roughly that the loop-free oriented matroids correspond to arrangements of generalized hyperplanes which are obtained from affine hyperplanes by certain topological deformations.

In Chapter VI arrangements of pseudolines are studied, and it is shown that they correspond to reorientation classes of simple orientable matroids of rank 3. Many examples are presented which, on the one hand, are not trivial but which, on the other hand, are simple to illustrate graphically. Moreover, some connections between oriented matroids and Grünbaum's exposition of pseudoline arrangements are described.

In Chapter VII constructions of oriented matroids are considered. It is demonstrated how oriented matroids can be extended, deformed, perturbed; and the connections between the old and the newly obtained matroids are discussed.

Chapter VIII is devoted to linear representability of oriented matroids. Topological representations are in some sense deformations of linear representations; while any oriented matroid admits a topological realization, there are many oriented matroids which cannot be represented over any field. To study linear representations, a certain semialgebraic variety $R(M)$ of an oriented matroid M is introduced, which is empty if and only if M is not representable.

In Chapter IX combinatorial properties of convex polytopes are studied by using oriented matroids. Any convex polytope induces an oriented matroid structure in a canonical way. Several new results on polytopes and new simplified proofs for known results are established.

Finally, in Chapter X it is shown that linear programming may be viewed as an oriented matroid problem. The oriented matroids yield a theoretical framework and in this way a better understanding for the combinatorics and the geometry of the simplex method for linear programs and their duals.

The authors intend to stimulate graduate students as well as researchers in various fields who want to concentrate on certain aspects of the theory of oriented matroids. Altogether the book is a very good achievement; however, in my opinion, two weak points should be mentioned. The first two chapters, which cover almost a hundred pages and should serve as a motivation of oriented matroid theory, are too long to motivate but too short to demonstrate the subject exactly. The student who is interested in oriented matroids as such might be demotivated by reading these two chapters if he is not acquainted with the related areas touched upon here. Thus, it would have been better to put these chapters at the end of the book as an appendix or to add a comment in the preface that the reader might begin by studying Chapter III. The second point is that the proofs of several theorems are postponed to subsequent chapters. Therefore, the reader who wants to understand parts of the theory in detail might have trouble.

To pass over to the positive aspects, it should be remarked that numerous exercises are included at the end of each chapter; thus the book is quite good literature for the student. Moreover, many figures are included. The five authors, all of whom are experts in oriented matroid theory, have succeeded in choosing common notation and conventions. Finally, the book contains a long list of references covering all essential papers published in oriented matroid theory thus far, as well as a few unpublished notes. Therefore, anybody who wants to study oriented matroids will not only find a compact survey concerning the theory but also a standard list of references.

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