perhaps with a very different perspective than someone previously exposed to such material.

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Catalan's conjecture, by Paulo Ribenboim. Academic Press, New York, 1994, xv + 364 pp., $64.95. ISBN 0-12-587170-8

Catalan conjectured in 1844 that the only solution to the equation

$$(*) \qquad x^p - y^q = 1$$

in integers x, y, p, q, all > 1, is given by $3^2 - 2^3 = 1$; in other words, 8 and 9 are the only consecutive powers. As Dickson pointed out in his famous *History of the theory of numbers*, in the Middle Ages Levi ben Gerson had solved the case $x = 3$, $y = 2$, and in 1738 Euler had solved the case $p = 2$, $q = 3$. Catalan himself contributed little, essentially only a simple remark on $x^y - y^x = 1$. Nevertheless, the conjecture gained considerable notoriety, and it became plain that it presented a challenge to number theorists somewhat akin to Fermat's Last Theorem.

As with the Fermat problem, factorization techniques over the cyclotomic and other fields have shed light in particular instances. Thus in 1850 V. A. Lebesgue dealt with the case $x^p - y^2 = 1$, and in 1964 Chao Ko treated the more difficult example $x^2 - y^q = 1$; this included an earlier theorem of S. Selberg with $p = 4$. The equations $x^3 - y^q = 1$ and $x^p - y^3 = 1$ were successfully resolved by Nagell in 1921, and the work led to valuable advances, notably by Ljunggren, on related equations such as $(x^p - 1)/(x - 1) = y^q$. Moreover, in 1961 Cassels obtained a particularly striking result in this context; namely, if p, q are odd primes, as one can assume, then $(*)$ implies that p divides y and q divides x. As Makowski noted, this shows that we cannot have three consecutive integer powers.

One of the most remarkable applications of the theory of linear forms in logarithms has been the effective determination of an explicit bound for all solutions x, y, p, q of $(*)$. The result depends on estimates for a nonvanishing expression

$$\Lambda = b_1 \log \alpha_1 + \cdots + b_n \log \alpha_n,$$

where the b's are integers and the α's are algebraic numbers. The original work of the reviewer of 1966 furnished a lower bound for $\log|\Lambda|$ that varied as a power of $\log B$, where B is the maximum of the $|b|$'s. This led to a new and effective resolution of the Thue equation whence, on viewing $(*)$ as superelliptic and reducing to Thue type, it followed that all solutions x, y are bounded by an explicit expression involving only p and q. The really decisive step came in the early 1970s with the Sharpening series of papers; here it was shown that the dependence of the lower bound for $\log|\Lambda|$ on A, the maximum of the heights of the α's, could be taken as $\log A$, which is best possible. In 1976, in an ingenious and celebrated memoir, Tijdeman used this result to demonstrate that all p, q in $(*)$ are bounded by an absolute constant and hence that the Catalan problem is, in principle, decidable. In practice, the quantities that arise from the work are too large, at present, to enable the conjecture to be resolved by machine computation. Nevertheless, considerable progress is being made in this respect. In particular, Glass et al. of Bowling Green State University in Ohio indicate that, through multiplicity estimates on group varieties and Laurent-type determinants, they can obtain a bound for $\max(p, q)$ below 10^{19} and a bound for $\min(p, q)$ below 10^{13}. Moreover, in the opposite direction, Mignotte, using theorems of Inkeri that imply certain congruences of the kind $p^{q-1} \equiv 1 \pmod{q^2}$, has shown that any nontrivial p, q must be at least 10^2. There is indeed some prevailing optimism that the problem will be completely solved quite soon.

The book by Ribenboim under review begins with an amusing preface, and it continues in a lively style. The volume is in fact the first to be devoted entirely to the subject, and, as such, it is especially welcome. The reader is taken along an essentially chronological path with lots of short detours into the surrounding countryside to maintain interest. We come upon, for instance, an introduction to theorems of Størmer on equations of Pellian type, an account of attempts to calculate decimals of π, and a brief excursion into powerful numbers and their distribution. However, the main strength of the book is undoubtedly its comprehensive treatment of all the main classical results on $(*)$ with full and accessible proofs.

The text has two distinct facets corresponding, roughly speaking, to the state of Diophantine analysis before and after the advent of transcendence theory. Formerly it was an inventive but largely incoherent mixture of ad hoc techniques often utilizing the detailed arithmetic of specific number fields of low degree. Now there is a general theory covering wide ranges of examples, and each numerical case is amenable to a complete solution by a combination of logarithmic form estimates and modern methods of computation. The author succeeds in imparting a very good impression of the first aspect but is plainly less at home with the second. Thus, on page 178 he lists three fundamental questions on binary cubic forms and proceeds "to summarize the most important results obtained thus far." In fact, we get some fourteen pages largely on the works of Delone, Nagell, etc., dating back to the 1920s, and it is not until the last half-page, almost as an afterthought, that we learn "recently, practical methods have been devised to solve Thue's equation." An example is briefly recorded due to Tzanakis and de Weger, but clearly there is no real appreciation at this point of the general theory. Later remarks, beginning on page 245, indicate a better understanding, but it is all very disconnected. The book by Shorey and Tijdeman entitled *Exponential diophantine equations* (Cambridge Univ. Press, London, 1986) covers somewhat similar ground and is more focused in this respect; it is recommended as additional reading. (Incidentally, the excessively brief and uninformative review of Shorey-Tijdeman in Bull. Amer. Math. Soc. (N.S.) **25** (1991), 145–146, is especially astonishing since it was here that, in a sense, the authors created the subject of the title.)

Looking at the text as a whole, there is no doubt that Ribenboim is greatly to be commended for bringing into the limelight an exciting research topic. The volume makes a nice companion work to his excellent 13 *Lectures on Fermat's Last Theorem* (Springer, New York, 1979).

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Non-classical elastic solids, by M. Ciarletta and D. Iesan. Pitman Research Notes in Mathematics, vol. 293, Longman Scientific & Technical, Harlow, Essex, 345 pp., $74.95. ISBN 0-582-22716-X

Let X_A denote a point in a body, and suppose the body deforms and the point moves to x_i. The deformation gradient F_{iA} is defined by

$$F_{iA} = \frac{\partial x_i}{\partial X_A} = x_{i,A}.$$

A classical elastic solid is one for which the stress tensor σ_{rs} and the internal energy U depend on $x_{i,A}$ and possibly X_A, i.e.,

$$\sigma_{rs} = \sigma_{rs}(x_{i,A}, X_A),$$
$$U = U(x_{i,A}, X_A).$$

In the linear approximation this gives σ_{rs} as a linear function of strain $x_{i,A}$, which is Hooke's law.

Over the past 30 years or so there has been significant interest in elastic-like materials which cannot be adequately described by the classical theory of elasticity. The present book is concerned with mathematical aspects of three theories which depart from classical elasticity theory.

A beautiful exposition of two of the nonclassical elastic solid theories may be found in Truesdell and Noll [6, p. 389]. They point out that Cauchy's second law in Continuum Mechanics is a constitutive assumption which says there are neither body couples nor couple stresses. A class of nonclassical materials are those for which there may be couple stresses or body couples present, and these are called polar materials; this theory was first developed by E. and F. Cosserat in 1907. In fact, it was Duhem who suggested including effects of direction via sets of points with vectors attached to them, thus giving rise to the theory of oriented media. This theory was developed by the Cosserats. Another generalization of classical elasticity is to elastic materials of grade 2 or higher, and this is also lucidly explained by Truesdell and Noll [6].

The theory of oriented media leads naturally to a theory of elastic rods (Antman [1]), or to elastic shell theory (Naghdi [3]). Also, it offers a very successful way to describe liquid crystals, a class of materials surely known to almost everyone in the developed world. Inclusion of body couples arises naturally in the industrially important field of ferrohydrodynamics (Rosensweig [5]). Here, the ferrofluid is a suspension of magnetic particles in a carrier liquid and