2. F. J. Almgren, Jr., and L. Wang, *Mathematical existence of crystal growth with Gibbs-Thomson curvature effects*, Preprint, 1994.
3. J. W. Cahn and J. E. Hilliard, *Free energy of a nonuniform system.* 1. *Interfacial free energy*, J. Chem. Phys. **28** (1958), 358–367.
4. S. J. Chapman, S. D. Howison, and J. R. Ockendon, *Macroscopic models for superconductivity*, SIAM Rev. **34** (1992), 529–560.
5. B. Derrida, J. L. Lebowitz, E. R. Speer, and H. Spohn, *Dynamics of an anchored Toom interface*, J. Phys. A **24** (1991), 4805–4834.
6. R. Dobrushin, R. Kotecky, and S. Shlosman, *Wulff construction: A global shape from local interaction*, Transl. Math. Monographs., vol. 104, Amer. Math. Soc., Providence, RI, 1992.
7. L. C. Evans and J. Spruck, *Motion of level sets by mean curvature.* 1, J. Differential Geom. **33** (1991), 635–681.
8. G. J. Fix, *Numerical simulation of free boundary problems using phase field methods*, The Mathematics of Finite Elements and Applications IV (Uxbridge, 1981), Academic Press, London and New York, 1982, p. 265.
9. M. Gage and R. Hamilton, *The heat equation shrinking convex plane curves*, J. Differential Geom. **23** (1986), 69–96.
10. M. A. Grayson, *The heat equation shrinks embedded plane curves to round points*, J. Differential Geom. **26** (1987), 285–314.
11. W. Kurz and D. J. Fisher, *Fundamentals of solidification*, third ed., Trans Tech Publications, Aedermannsdorf, Switzerland, 1989.
12. S. Luckhaus, *Solutions for the two-phase Stefan problem with the Gibbs-Thomson law for the melting temperature*, European J. Appl. Math. **1** (1990), 101–111.
13. W. W. Mullins, *Two-dimensional motion of idealized grain boundaries*, J. Appl. Phys. **27** (1956), 900–904.
14. O. Penrose and P. C. Fife, *Thermodynamically consistent models of phase-field type for the kinetics of phase transitions*, Phys. D **43** (1990), 44–62.
15. J. A. Sethian and J. Strain, *Crystal growth and dendritic solidification*, J. Comput. Phys. **98** (1992), 231–253.
16. W. A. Tiller, *The science of crystallization: Macroscopic phenomena and defect generation*, Cambridge Univ. Press, Cambridge and New York, 1991.
17. A. A. Wheeler, W. J. Boettinger, and G. B. McFadden, *Phase-field model for isothermal phase transitions in binary alloys*, Phys. Rev. A **45(10)** (1992), 7424–7439.

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The classification of the finite simple groups, by Daniel Gorenstein, Richard Lyons, and Ronald Solomon. Math. Surveys Monographs, vol. 40, no. 1, Amer. Math. Soc., Providence, RI, 1994, xiv+165 pp., $50.00. ISBN 0-8218-0334-4

This is an unusual book. It is the first volume in a projected multi-volume series, estimated by the authors to eventually total between three and four thousand pages. The goal of the series is to provide a second proof of one theorem: the classification of the finite simple groups. (Skeptics might prefer "first proof".) Specifically the authors propose to write down for the first time in one place a complete proof of the Classification, reducing its length by a factor of at least 2 or 3 by reorganizing the proof and taking advantage of modern group theoretic technology and a more direct route to their goal. This first volume surveys fundamental concepts and results underlying both the original proof and the new proof and outlines the new proof the authors are working toward.

An excellent article on the Classification appeared recently in the February 1995 issue of the *Notices* [1]. The article was written by Ron Solomon, one of the authors of the book under review. In it, Solomon sketches the original proof of the Classification and talks about the series of books he, Gorenstein, and Lyons are writing. (Sadly, Gorenstein died in 1992, but Lyons and Solomon continue with the project.) I refer all readers of this review to Solomon's article, and rather than repeating what Solomon has to say, I propose instead to try to place the Gorenstein-Lyons-Solomon (GLS) series in a context which will help explain its importance. I will, however, have a little to say about the portion of the original proof and the GLS revision that is most controversial.

Early in the seventies after a period of intense activity on finite groups, a plan for classifying the finite simple groups began to take shape. Within a few years that plan crystalized into a very specific outline, to the point that in 1976 most knowledgeable finite group theorists felt that a proof of the Classification was inevitable and would probably come within ten years. The perception that the Classification was nearly complete had a number of side effects which, in retrospect, were unfortunate. During the succeeding five years, as the proof was being completed, the flood of students that had been entering the field began to dry up, and many of the established mathematicians who had been instrumental in the program moved on to other problems and areas. Further, the work during this period was not subjected to the usual reworking and simplification that important new results usually receive. Much of this work was in the most poorly understood chapter of the Classification devoted to the small groups of even characteristic, a chapter we will return to later. Some of it remains unpublished to this day.

When the Classification was completed, the landscape of finite group theory was instantly altered. Virtually all the classic problems in the field, some over a century old, became easy corollaries to the Classification, and the knowledge of the simple groups developed as part of the Classification. The lifeblood of a mathematical specialty is exciting, visible problems. The apparent disappearance of such problems accelerated the exodus from the field, leaving few people interested in giving the proof of the Classification retrospective attention. Unfortunately it is just this sort of foundational reexamination that can lead to a more profound understanding of a subject, and there *is* room for improvement in our understanding of the finite simple groups. For example, our current description of the sporadic groups leaves something to be desired but could be improved if we could find a nice class of objects upon which to represent the sporadics which would supply a uniform description of these groups. Also, there are still unexplained mysteries which new points of view might clear up. For example, I don't feel we yet have a satisfactory explanation of the apparent connection between the sporadics, particularly the Monster, and modular functions of genus 0.

As might be expected, the Classification soon opened up new problems which were more interesting than the old classics (with the exception of the Classification itself). The new problems often required different skills, background, and even a different mind set than the old problems. Many of them came from other areas of mathematics. The new problems couldn't be solved without the Classification, but one could solve them without knowing anything about the *proof* of the Classification. Rather, a deep knowledge of the finite simple groups was required, particularly knowledge of subgroup structure and linear representations.

Today much of the interesting activity involving finite groups focuses on their representations in number theory, combinatorics, geometry, or topology. Purely group theoretic activity tends to involve the subgroup structure and linear representations of the simple groups, and even here some of the problems receiving the most attention are primarily of interest because of an application in another field of mathematics. The Classification is a prerequisite for all this activity, but by now few finite group theorists have a detailed knowledge of the proof of the Classification, and given the current state of the literature, it would be difficult for them to acquire that knowledge.

Hopefully you are now convinced that the GLS program is important. Because so many mathematicians are using the Classification but few understand its proof, it becomes imperative to write down in one place a complete, relatively self-contained proof of the result that can actually be read by those sufficiently well versed in finite group theory. With such a treatment, the mathematical community can feel confident about the validity of the theorem, or at least as confident as one can be about a difficult 3,000-page proof.

Having raised the specter of confidence, it is perhaps time to discuss the most problematic part of the GLS program, involving the small groups of even characteristic, which you may recall constituted the last chapter of the original proof and received the least attention. Difficulties arise because GLS do not plan to simply modify the first proof but instead plan to use some new techniques which began to appear shortly before the simple groups were classified. This machinery applies to groups generated by a family of subgroups with a common Sylow 2-subgroup, each of which has a generalized Fitting subgroup which is a 2-group. GLS hope specialists in these techniques will supply a new treatment of small groups of even characteristic. For this reason they have not yet written up their own version of this part of the proof.

Unfortunately a new treatment using these techniques has been expected for fifteen years. Even worse, for fifteen years important parts of the original work on the small groups of even characteristic have remained unpublished. It is time for finite group theorists to firm up this crucial part of the foundations of our subject.

Currently the psychology and sociology of mathematics is problem driven. Probably for the most part this is a good thing, but in this instance it had some unfortunate side effects. The Classification revolutionized finite group theory by changing the type of problems finite group theorists work on and the nature of the knowledge and machinery they use to solve problems. This change began before the Classification was complete, delayed the publication of a readable proof of the Classification, and encouraged group theorists to accept first solutions to foundational problems connected with the Classification that deserve more attention. It is my hope that the GLS project will undo some of this damage by stimulating others to improve portions of the proof in a more dramatic fashion and that the proof of the Classification will finally begin to receive the kind of second-generation attention which can lead to a better understanding of the simple groups.

References

1. R. Solomon, *On finite simple groups and their classification*, Notices Amer. Math. Soc. **42** (1995), 231–239.

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Mathematical Go: Chilling gets the last point, by Elwyn Berlekamp and David Wolfe. AK Peters, Wellesley, MA, 1994, xx + 234 pp., $34.95. ISBN 1-56881-032-6; also in paperback as *Mathematical Go endgames: Nightmares for the professional Go player*, Ishi Press International, San Jose, London, and Tokyo, $24.95. ISBN 0-923891-36-6

Last July, at the Combinatorial Games Workshop in Berkeley, David Blackwell suggested that most of mathematics may be chaotic, and that it is only the small part where we recognize patterns that we actually call mathematics. Combinatorial game theory tends to foster such a view. Are games with complete information and no chance moves of little interest because there are pure winning strategies? On the contrary, when we come to analyze even the simplest of such games, we often run into regions of complete chaos.

David Gale wrote in [4] that he had barely scratched the surface of combinatorial game theory, but

> the experience has left me with an overwhelming sense of awe at the unfathomable diversity of mathematics itself. The authors of [1] have created a mathematical fairyland ... [some] people say all of mathematics is really one ... I wonder if this belief in ultimate unity may not be just wishful thinking ... My own hunch is that mathematics (perhaps physics too) is not going to unify ... Combinatorial game theory is just one example of what unfettered mathematical imagination is capable of creating ... There will be many others ... Mathematics will continue to diversify in totally unpredictable ways ...

But now this fairyland turns out to be part of the real world. Go is the most significant of combinatorial games, whether you measure by popularity, playability, or resistance to computer attack. Yet Berlekamp, a mere 10-kyu (connotes "child") player, has been to Japan, set up endgame positions against