BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 33, Number 2, April 1996

Algebraic curves and Riemann surfaces, by Rick Miranda, Graduate Studies in Math., vol. 5, Amer. Math. Soc., Providence, RI, 1995, xxi+390 pp., $59.00, ISBN 0-8218-0268-2

The subject of Riemann surfaces is endlessly fascinating and remains a remarkably active and challenging area of current research despite its relatively long history. Although various introductory and survey monographs on Riemann surfaces have appeared in the past ten to twenty years, the subject is sufficiently broad and active that there appears always to be room, indeed possibly always to be a need, for yet another one. The book by R. Miranda has a perspective and charm that make it an excellent addition to the survey literature on the subject.

The book is basically a leisurely and well-presented introduction to algebraic geometry through the study of algebraic curves over the complex numbers. Although the emphasis is on the algebraic geometry, the underlying complex manifold is always kept firmly in view, and various techniques and tools of complex analysis are used freely; so the book is an introduction to Riemann surfaces as well. It contains an abundance of examples and problems and develops the basic notions, such as functions, divisors, differential forms, projective imbeddings, maps between curves, thoroughly and carefully; consequently it is excellent for self-study by beginners in the field. While it may be a bit leisurely for those who already have a reasonable knowledge in this area, it nonetheless repays examination by anyone interested in the field for some interesting insights and for a number of excellent ideas about the development and presentation of the material.

Since the focus of the book is on algebraic geometry, it is assumed from the beginning that the Riemann surfaces being considered possess enough nontrivial meromorphic functions—that is, that they are algebraic curves. With this having been assumed, the Riemann-Roch theorem is developed through an analysis of the problem of finding meromorphic functions on the surface with specified principal parts. The Riemann-Roch theorem is then applied to obtain various properties of algebraic curves themselves: a classification of curves of genus at most five; some properties of special divisors, such as Clifford's theorem and Castelnuovo's bound; relations between the genus, degree, and imbedding dimension of algebraic curves; Weierstrass points and some of their uses. This is followed by a fairly brief introduction to Abel's theorem and the Jacobi variety. The book then leaves the more mundane if not classical areas of algebraic curves to introduce the readers to sheaves and sheaf cohomology, reinterpreting some of the earlier discussion in the light of this machinery to illustrate some of the uses of these tools. The last quarter of the book thus has a somewhat different character from the first part: it is less an introduction to the subject of algebraic curves or Riemann surfaces proper, more an introduction to the techniques of sheaves in algebraic and analytic geometry, with the examples of algebraic curves to motivate and illustrate these techniques.

In so many ways I found this a charming book, and one that I would happily recommend both to those advanced undergraduates who have an interest in this area and to any graduate students who wish to learn more about this important and lively area of mathematics; many graduate students interested in algebraic geometry could profit by an introduction to the more abstract techniques of algebraic geometry through concrete applications to curves as discussed in this book. Both beginners and experts as well will find a number of fascinating topics that do not normally appear in introductory texts, such as a very nice discussion of Riemann's count of the moduli for Riemann surfaces, which is approached both classically through the tally of the parameters describing branched covers of the Riemann sphere in the elementary part of the book and in a thoroughly current way through a discussion of deformations of complex structures in the more advanced part of the book.

R. C. Gunning
Princeton University