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Introduction to mechanics and symmetry: A basic exposition of classical mechanical systems, by Jerrold E. Marsden and Tudor S. Ratiu, Texts Appl. Math., vol. 17, Springer-Verlag, New York, 1994, xiv + 500 pp., $64.50, ISBN 0-387-97275-7

Classical mechanics motivated Newton's development of calculus in the latter half of the seventeenth century and developed along with it through much of the succeeding two centuries. One has only to recall the names of the great mathematicians and physicists of those eras to recognize how closely the analysis of the seventeenth, eighteenth, and nineteenth centuries was intertwined with mechanics: Newton, the Bernoullis, Euler, Lagrange, Legendre, Laplace, Poisson, Liouville, Dirichlet, Gauss, Riemann, Jacobi, Hamilton, and, of course, Poincaré. In fact, it wasn't until the nineteenth century that distinctions began to be made between mathematicians and physicists. Many of the early practitioners were known simply as natural philosophers. Even in the nineteenth century the distinction between a "geometer", an "analyst", and a "natural philosopher" was by no means as clear-cut as it is today. Up until our own century classical mechanics was regarded as an integral part of the curriculum of every student of mathematics. However, as other trends gained sway in mathematics, mechanics was pushed more and more to the periphery, until the present state was reached, where mechanics is hardly taught in mathematics departments and is not taught in great depth in physics departments (though there it is at least still considered a central topic in the undergraduate and graduate curricula). In physics the focus shifted to studying classical mechanics only to the extent that it serves as a basis for the "more interesting" studies of the quantum mechanics of atoms and molecules and of the solid state.

While the early part of the twentieth century did see some notable achievements in classical mechanics, for example, the elaboration of many of Poincaré's seminal ideas by Birkhoff and others, and the development of the celebrated KAM theory by Siegel, Kolmogorov, Arnold, and Moser, for the most part the interests of mathematicians lay elsewhere. However, in recent years much has changed. In the meantime mathematicians had been developing Poincaré's vision for understanding mechanical systems qualitatively, under the general heading of dynamical systems. Though this development in some ways strayed from the central topics of Hamiltonian systems and celestial mechanics, which had maintained their ascendancy through the time of Poincaré and Birkhoff, in many ways it is responsible for the current revitalization of the subject. With the new tools and points of view that were necessary in understanding general dynamical systems, mathematics reached a point where some of the old problems could be attacked with renewed vigor and new questions could be asked. Other factors influencing these shifts in emphasis have been the advent of the modern high-speed computer, which has made it possible to examine extensively many quite complicated dynamical systems "experimentally", and developments in modern technology, such as plasma physics and the detailed study of the dynamics of molecular systems, which have given new impetus to our desire to understand certain rather elaborate classical mechanical systems in a qualitative (and even quantitive) way. In the latter case classical mechanics enters

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because the nuclei in molecular systems are found to evolve in close accord with the quasiclassical limit. In particular, when the quasiclassical limit has complicated or unstable dynamics, the corresponding molecular dynamics tend to exhibit surprising behaviors. Here one finds range for the KAM theory and the developments associated with strange attractors, Smale horseshoes, structural stability, Liapunov exponents, symbolic dynamics, turbulence, and routes to chaos.

Modern books attempting a comprehensive coverage of classical mechanics together with its attendant mathematical structures such as manifolds, tangent spaces, the Legendre transform, phase space, symplectic geometry, etc., include the books of Abraham and Marsden [1], Arnold [2], and Thirring [12]. A very nice survey of the fields of classical and celestial mechanics from the modern point of view can be found in [4]. Other treatments with similar viewpoints but more modest scope include Mac Lane's Chicago lecture notes [7] and Loomis and Sternberg [6]. More specialized topics are treated in the books by Arnold and Avez [3], Moser [9], Siegel and Moser [10], and Sternberg [11]. For a rather different, but very interesting, treatment of mechanics, the reader is encouraged to consult Gallavotti [5]. Truesdell's essays [13, 14] on many of the early advances in mechanics and the people involved also provide stimulating and thought-provoking reading.

The book under review here is rather more modest in scope (after all, one of its authors already coauthored the comprehensive book [1] mentioned above). As its title states, it concentrates on mechanics and symmetry. One of its strengths is that it begins by introducing several concrete physical models to which it returns repeatedly as new ideas and formalisms are introduced. This should make it accessible and useful to students in mathematics, physics, and engineering. Along with the necessary material and standard topics such as the Lagrangian and Hamiltonian formalisms and Poisson brackets, the book gives a brief introduction to Lie groups and treats such topics as variational theory, resonances, geometric phases, holonomy and its relation to control theory, bifurcation and stability, normal forms, the energy-momentum method, momentum maps, and reduction theory (for use in the presence of symmetry). In fact, the main thrust of the book is toward analyzing mechanical systems with symmetry. For example, for a system with an SO(3) symmetry, one often wants to analyze the stability of a uniformly rotating state. This can be accomplished in a consistent and uniform fashion by means of reduction theory as developed in detail in the book under review. This will also be the focus of a follow-up volume (see below). As an illustration of its concrete flavor, the book ends with a 30-page chapter on "The Free Rigid Body", which applies in an instructive way many of the abstractions developed in the earlier chapters. This is very much a modern treatment of rigid body motion not to be found in the physics books, but nonetheless well worth the effort on the part of a student of physics or engineering. There are also many welcome historical comments scattered throughout the text.

I found very few errors in my skimming of the book, and none more than small typos. Marsden's earlier volume [8] might be regarded as a companion volume to this one, one that can serve to whet the reader's appetite for the more comprehensive treatment contained here. The potential reader might also like to know that there is a projected second volume on reduction theory and its applications in the works. These books should provide ready access to the current, and rapidly expanding, research in classical mechanics, particularly those in which the authors and their coworkers have made extensive contributions. In particular, the book under review would be a fine addition to the bookshelf of anyone with an interest in recent developments in mechanics, particularly those interested in using reduction procedures based on symmetries to understand concrete physical systems.

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