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Exact controllability and stabilization; the multiplier method, by V. Komornik, Res. Appl. Math., vol 36, Wiley-Masson, 1994, viii + 156 pp., $39.95, ISBN 0-471-95367-9

Professor Komornik's work is concerned with the related mathematical theories of controllability and stabilizability in the context of elastic and electromagnetic systems modelled by linear partial differential equations such as the wave equation, Maxwell's equations and the (biharmonic) plate equation. The controls appear as inhomogeneous terms in the boundary conditions applying on a subset, Γ_1, of the boundary, Γ, of the geometric region, Ω, supporting the physical system under study.

A typical control question is that of *null controllability*; given an initial state consisting of a displacement $w_0(x)$ and a velocity, $v_0(x)$, $x \in \Omega$, and assuming these functions to lie in an appropriate (Hilbert) state space, is it possible, for a given $T > 0$, to find a control function $u(x,t)$, $x \in \Gamma$, $t \in [0,T]$, and lying in an appropriate (Hilbert) control space, such that the resulting solution, $w(x,t)$, $x \in \Omega$, $t \in [0,T]$, satisfies null terminal conditions $w(x,T) \equiv 0$, $v(x,T) \equiv \frac{\partial w}{\partial t}(x,T) \equiv 0$, $x \in \Omega$? An example of application occurs in connection with the suppression of acoustic vibrations in an enclosed cavity, such as an aircraft cabin, through use of sound-cancelling acoustic transducers mounted on portions of the cavity wall.

A great deal of mathematical energy has been expended on problems of this sort. The reviewer considered certain aspects of these problems in the context of the wave equation in the late 60's and early 70's, culminating in the observation that, for these time reversible processes, the existence of controls in feedback form leading to uniform exponential decay of system energy is equivalent to finite time controllability, posed in the "finite energy" state space. At the same time it was realized that these control problems were intimately connected with exterior scattering problems for the wave equation, as explored around the same time by C. S. Morawetz, W. A. Strauss and others. In attempting to make use of this work in the control context it became clear that one of the main difficulties derived from geometric complications in the energy decay estimates, involving the shape of the domain Ω, and the fact that the standard Sobolev trace theorems, as notably developed by J.-L. Lions and E. Magenes in their landmark two-volume work, did not by themselves provide sufficient regularity properties of the restrictions of solutions $w(x,t)$ to the boundary, Γ, to allow extraction of controls $u(x,t)$ in the required control spaces.

Many of the geometric difficulties were treated by G. Chen during the late 70's and early 80's. Further, C. Bardos, G. Lebeau and J. Rauch used methods of geometrical optics in the scattering context to obtain definitive energy decay and control results for the wave equation. Sophisticated *multiplier methods* were developed by J. Lagnese, I. Lasiecka, R. Triggiani and others to resolve these questions in another way; this method also proved adaptable to other systems, such as the

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Kirchhoff plate equations. Lasiecka and Triggiani largely resolved the difficulties associated with the standard trace estimates, showing that solutions possessed *hidden regularity* of a higher order than predicted by those estimates.

It is in this historical context that Professor Komornik's book, which grew out of a series of lectures given in France, Hungary and the United States, needs to be understood. For, while always generous in attribution, Komornik does not provide much of the historical and technological background in this concise monograph. In this work he concentrates on proofs of controllability and stabilizability which can be characterized as *optimal*, both with regard to the control time, T, where that is pertinent, and with regard to the economy of argument and exposition.

The work begins with a study of exact controllability problems by the Hilbert Uniqueness Method (HUM), which connects those problems to certain questions of boundary observability of solutions of the corresponding homogeneous (i.e., unforced) adjoint systems. In this part of the work he replaces some older, indirect, compactness - uniqueness arguments by constructive proofs. Further, he considers equations differing from the standard examples by inclusion of lower order terms and rather general sets of boundary conditions, in some cases simplifying results obtained previously with the standard boundary conditions. Subsequently he undertakes the development of a general and constructive approach to the improvement of earlier estimates of the (minimal) exact controllability time, using an estimation method due originally to A. Haraux.

The second part of the book is devoted to stabilizability theory. Using a modified and simplified Liapounov approach, strong and uniform boundary stabilization theorems are obtained, as well as a long overdue re-statement of the principle connecting controllability to bi-directional stabilizability referred to earlier. The multiplier method, applied systematically throughout the book, is seen to be remarkably elementary and economical.

Issued as a very compact viii + 156 page volume, including a valuable set of references, Professor Komornik's mini-treatise is well worth the $39.95 investment and deserves an honored place on the applied mathematician's bookshelf.

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