BOOK REVIEWS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 36, Number 1, January 1999, Pages 95–97
S 0273-0979(99)00767-3

Jacques Hadamard, a universal mathematician, by V. Maz′ya and T. Shaposhnikova (translated and edited by P. Basarab-Horwath), American Mathematical Society and London Mathematical Society, Providence, RI,1998,xvii+574 pp., $63.00 ($47.00 to members), ISBN 0-8218-0841-9

I

Some mathematicians are well remembered without being well acknowledged: the name circulates, but the contributions which caused the fame are largely forgotten or at least not distinguished from others'. An outstanding case for this century is Jacques Hadamard (1865–1963): around sixty results or concepts to which his name is attached (quite often correctly), and yet the historical record of his achievements is remarkably little recorded. After his death a measure of tribute appeared, more in Switzerland than in his native France; since then he has received little attention.

All the more reason then to welcome this fine and exhaustive book, which treats in detail both his exceptionally long life and comparably important work. It is, in fact, divided into two parts this way, but with limited polarity, since quite a few technical details are given in the biographical part. Of the many illustrations presented, a good proportion portray or concern Hadamard and include even magazine and newspaper articles; this is a fine aspect of the book, especially as Hadamard lost most of his manuscripts in the occupation of his flat in Paris during the Second World War (p. 20: located sources are listed on pp. 559–561). Most of the other illustrations portray other mathematicians to whom his work relates, but they are rather overdone, especially with the mediocre reproduction. It is hardly necessary to show, for example, Newton and Leibniz, especially when caught in fogs (pp. 366–367), and the grounds for including Joseph Bertrand (p. 332) are not clear anyway.

The quality of the book is greatly strengthened by very full bibliographies, in three parts: (i) Hadamard's own publications (contributions to the *Intermédiare des mathématiciens* of the 1890s are missing), (ii) writings on him, and (iii) everything else. However, the manner of citing is uninformative, as an item is located only by the appropriate roman number above followed by an Arabic ordering numeral; thus, for example, '[I.334]' denotes a paper by Hadamard (actually the same as [I.224]). A system such as 'Hadamard 1913b' or 'Darboux 1878' would have given

1991 *Mathematics Subject Classification.* Primary 01-XX; Secondary 11-XX, 26-XX, 30-XX, 35,-XX, 46-XX, 53-XX, 73-XX, 76-XX.

the narrative a sense of time and source which is missing in places. The index is excellent on names but rather sparse on subjects.

II

The survey of Hadamard's life begins with a good account of his training at the Ecole Normale Supérieure (ch. 1). But his relations with other mathematicians come through less clearly. In particular, around the turn of the century he and his *normalien* colleague Emile Borel (1871–1956), whom he influenced (pp. 75–78), were leading a new school in set theory and mathematical analysis; but while Borel was at the centre of a constellation including Baire, Lebesgue and Fréchet, Hadamard seems to have been comparatively alone, although enjoying good personal relations. An examination of his position in this strong mathematical community deserves further scrutiny. One important common factor was their use of set theory, where the community was very influential in its general acceptance in the late 1890s. Hadamard made a point of emphasising its merits at the first International Congress of Mathematicians in 1897; his paper is even photo-reproduced (p. 397), but it would have been worth mentioning that he misdefined well-ordering in it!

Among other contacts, Hadamard's honorary doctorate from Göttingen University in 1899 (at a remarkably early age) is recorded (p. 89), but not the occasion: the inauguration of the Gauss-Weber statue, when David Hilbert presented his famous book on the foundations of geometry.[1] This event tightens the contact with Hilbert (p. 91: Hilbert's wife was actually called 'Käthe'). Again, his contributions to operational research (pp. 245–249, a little-known part of his work) in England during the Second World War probably links to his relationship with P. M. S. Blackett (p. 291).

III

Hadamard's mathematical work comprises quintets of textbooks and of research monographs, and nearly 400 papers and articles (on all subjects). The output is especially astonishing when one learns of his way of working: apparently disorganised and unwilling to write systematically, he dictated text to his wife Louise (1868–1960); the performance included the speech act 'poum', of which each occurrence indicated a unit of line-length to be left blank for him to write in a formula later (pp. 107–109). This strange method was successful thanks to another of his gifts: a strong memory.

Much of Hadamard's research centred on real- and complex-variable analysis, with a special focus on differential equations (ch. 9) and the calculus of variations (ch. 11), with applications of the findings to mechanics and mathematical physics (ch. 12). Among his finest results were surprising properties of trajectories, found in the late 1890s (pp. 354–360); a noteworthy recent influence, not mentioned here, is on Karl Popper, who used paper [I.62] on trajectories along geodesics to back up his own arguments for indeterminism and the epistemological status of initial conditions in scientific theories ([2], 39–40).

[1] Among his many other awards, Hadamard was elected Foreign Member of the Royal Society of London in 1932, with G. H. Hardy as initiator, but curiously few other mathematician members supported the proposal (election certificate, Society Archives).

Hadamard stressed the notion of 'well-posedness', in effect a metamathematical appraisal of problems to ensure that an answer can be meaningful. While all mathematicians think this way (or should) to some extent, he handled it in a far more sophisticated manner and clearly aided the quality of his work on differential equations. It shows his natural philosophical sense, which came out in another non-standard way with his interest in the psychology of mathematical thinking. The best-known outcome is his book of 1945 (pp. 489–495, including previously unpublished responses to Hadamard's questionnaire by Louis de Broglie), but concern with heuristics went back to the 1900s [I.127].

Among other contributions is Hadamard's famous proof of the prime number theorem, in 1892 (pp. 318–321), which established his fame (and maybe his honorary doctorate later in the decade). Next year he found an important inequality for determinants, at a time when that branch of mathematics was still not standard fare (pp. 383–388). Indeed, his range is impressive.

IV

The authors cover a remarkable amount of Hadamard's output and also his concerns outside mathematics: family life (he lost two sons in the First World War and the other one in the Second), a deep interest in botany, concern with the victimisation of his distant relative Alfred Dreyfus, and pacifism. It is a pity that the publishers tell us nothing about the authors. One learns from their text that they are Soviet-born mathematicians, husband and wife, now resident in Sweden. Their book is an elaboration of a Russian biography of Hadamard published in 1990 (item [II.52]), which Shaposhnikova wrote with the late E. M. Polishchuk. He also published a useful biography of Borel [1]. Now *there* is another member of that community deserving a detailed treatment; can history of history repeat itself?

References

[1] E. M. Polishchuk (1980), *Emil′ Borel′ 1871–1956*, Moscow (Nauk). MR **82g:**01067
[2] K. R. Popper (1982), *The open universe*, London (Hutchinson).

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