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Topics in disordered systems, by C. M. Newman, Birkhäuser-Verlag, Basel, 1997, vii + 88 pp., $22.50, ISBN 3-7643-5777-0

In statistical mechanics a group of important models are of the following type. A configuration is given by an assignment of a local state s_x to each site x of the integer lattice $\mathbb{Z}^d$. Physically, a choice of local state might represent a species of particle or (the orientation of) the magnetic moment of an atom. The energy associated to the configuration s is given by a Hamiltonian

$$H(s) = -\sum_{\{x,y\}} J_{\{x,y\}} h(s_x, s_y) \tag{1}$$

where the sum is over all bonds (i.e. unordered nearest-neighbor pairs of lattice sites) $\{x, y\}$. Here $h(s_x, s_y)$ gives the form of the pair interaction between adjacent sites, and $J_{\{x,y\}}$ gives the strength (with sign + or −) of that interaction. In accordance with Boltzmann's Principle, the probability of the configuration s is given by a Gibbs distribution:

$$P(s) = Z^{-1} e^{-\frac{1}{T}H(s)}, \tag{2}$$

where T denotes the temperature and the normalizing constant Z is the partition function. Equations (1) and (2) should be considered formal only; to make them precise one must first restrict to a finite subregion Λ of the lattice and impose a boundary condition, then let $\Lambda \nearrow \mathbb{Z}^d$. Infinite-volume Gibbs states are corresponding subsequential limits of finite-volume Gibbs distributions.

The structure of the (convex) set of Gibbs states is of fundamental importance. For typical systems one expects that there will be a unique Gibbs state when the temperature is sufficiently high, and multiple Gibbs states when the temperature is sufficiently low. For simple systems such as the homogeneous ferromagnetic Ising model, in which $s_x \in \{-1, 1\}$, $h(s_x, s_y) = s_x s_y$ and $J_{\{x,y\}} \equiv J > 0$, this picture is fairly well understood, if in some parts only heuristically. In two dimensions, for example, there are only two extremal Gibbs states, one dominated by +1 and the other by −1. This book, in contrast, concerns *disordered systems*, in which the signed interaction strengths $\mathcal{J} = \{J_{\{x,y\}}, x, y \text{ nearest neighbors}\}$ are themselves random, typically independent and identically distributed; the probability in (2) is then conditional on $\mathcal{J}$. (There are other types of disorder not treated in the book, such as random site-dependent external fields, bond and site dilution, annealed disorder and Hopfield models.) Disordered systems arise physically in many ways (see [FHbk], [My].) Certain alloys mix ferromagnetism with antiferromagnetism. The RKKY interaction ([Ka], [RK], [Yo]) is an oscillating function of the random distance between atoms so is effectively random with a symmetric distribution. Crystals contain random impurities. A monolayer of atoms may lie on a randomly varying substrate.

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Many spin systems exhibit long-range order at low temperatures; correlations between local states do not decay to 0 as the separation grows. In the (free-boundary) ferromagnetic Ising model, for example, one sees a global predominance either of $+1$ or of -1 with probability $1/2$ each. *Spin glasses* are materials in which the configuration becomes "frozen in" below some critical temperature, but without the appearance of an obvious long-range order. This can occur when the material incorporates both ferromagnetism and antiferromagnetism randomly. Chapters 2, 3 and 4 are concerned with the *Edwards-Anderson (EA) spin glass model* [EA] (and variants thereon), one of the main lattice models for spin glass behavior. It is a disordered Ising model in which the interactions $J_{\{x,y\}}$ are symmetrically distributed. Here the structure of the set of Gibbs states may be complex and is not well-understood, even nonrigorously by physicists, who typically subscribe to one of two mutually incompatible heuristics. The first heuristic, based on scaling arguments, is due to Fisher, Huse, McMillan and others ([FH], [Mc]) and predicts that at low temperatures there are almost surely exactly two extremal Gibbs states, differing by a global flip of all spins and depending nontrivially on $\mathcal{J}$. The second heuristic is based on analogy to better-understood heuristics for the *Sherrington-Kirkpatrick (SK) model* [SK], a mean-field analog of the EA model in which there is no spatial structure and every pair of sites interacts. The SK analysis by Parisi and others ([MPV], [Pa]) suggests that at low temperatures there are almost surely infinitely many extremal Gibbs states, again with nontrivial dependence on $\mathcal{J}$.

The drawing of analogies between the EA and SK models is fraught with peril, however. For example, there is no infinite-volume formulation of the SK model; there are only limits of various quantities as the number of sites $n \to \infty$. Hence there is no precise notion of an (extremal) Gibbs state. Physicists are free to ignore such peril, but mathematicians seeking rigor must concern themselves with such questions as: what is the proper formulation in the (infinite-volume) EA model of the results "known" nonrigorously in the limit $n \to \infty$ for the (finite-volume) SK model? One of the main contributions of Newman and D. L. Stein ([NS92], [NS96a], [NS96b]) has been to propose or rigorously rule out some possible answers to this and related questions; this material is presented in Chapter 4.

For the high-temperature regime the situation is somewhat better. In Chapter 3 Newman presents some of his results from [Ne] in which he uses percolation methods to give sufficient conditions for uniqueness of the Gibbs state. This involves an adaptation to the disordered case of the Fortuin-Kasteleyn (FK) random cluster model of [FK]. Some familiarity with the standard FK model would be helpful to the reader here, though the material is, strictly speaking, self-contained.

A *ground state* (in infinite volume, or in finite volume with specified boundary condition) is a configuration which locally minimizes the energy; that is, no local change decreases the energy. In many models, at very low temperatures a typical configuration looks like a ground state with local fluctuations. Thus understanding ground states is an important step toward understanding the structure of the Gibbs states. For the EA spin glass model, a configuration s is clearly a ground state if $J_{\{x,y\}}$ and $s_x s_y$ have the same sign for all $\{x, y\}$. But for typical $\mathcal{J}$ there is no such s; this is the phenomenon of *frustration*, and its consequence is a complicated set of ground states. In Chapter 2 Newman characterizes finite-volume ground states under the condition that the interactions $J_{\{x,y\}}$ be extremely variable, in the sense that with high probability, for each fixed x, if the magnitudes $|J_{\{x,y\}}|$ are put in decreasing order, then each one exceeds the sum of all smaller ones; this is a sort of "low-temperature" condition on the configuration $\mathcal{J}$. When one tries to extend to infinite volume, this characterization leads to some interesting open questions about invasion percolation and minimal spanning forests.

Chapter 1 concerns the ferromagnetic case, that is, disordered Ising models with nonnegative interactions $J_{\{x,y\}}$. Here, in two dimensions, in any ground state, if there is an infinite interface between a region of $+1$ and a region of -1, this interface must follow a doubly infinite geodesic for a first-passage percolation model derived from the random variables $J_{\{x,y\}}$. More specifically, $J_{\{x,y\}}$ is treated as the random length of the bond which perpendicularly bisects $\{x, y\}$ when the integer lattice is shifted by $(1/2, 1/2)$, and geodesics are paths which are everywhere locally of minimal length. A number of fundamental problems regarding geodesics in first-passage percolation remain unsolved—do doubly infinite ones exist? The likely answer seems to be no; the progress made thus far by Newman and others ([LN], [We]) is presented. Other properties of these geodesics are of interest in the context of boundary roughness in growth models [LNP], another area in which physicists do not even agree on the proper heuristics [KS].

While there are several excellent treatments covering the central principles of rigorous statistical mechanics, including the books of Georgii [Ge] and Simon [Si], there is an unfortunate paucity of books covering the enormous work over the last 35 years on specific models, even for the Ising model, which is probably the most well-studied; this can be a hindrance to researchers wishing to enter the field. This paucity makes the present book particularly valuable. The book is accessible to those with an understanding of the basic theory of Gibbs distributions, yet (partly because so little is rigorously understood!) it proceeds almost immediately to the frontiers of research. It is fresh—most of the material presented is 6 years old or less, and almost none of it previously appeared in book form. The book is a summary of the considerable progress so far in the research program of Newman and his collaborators; as such it is not an attempt to survey the entire field of disordered systems, but it serves as an excellent introduction nonetheless. (Another recommended reference is the recent collection [BP] of surveys on spin glasses, covering a wider range of models than Newman's book, particularly the Hopfield model, but with perhaps less introductory and background material. It has more emphasis on mean-field models, and necessarily less comprehensiveness on any one model. This does not overlap with Newman's book, except in the article in [BP] by Newman and Stein. For a broader introduction to disordered systems, see [Fr].) Newman's well-written, cohesive presentation should be of great assistance to new and experienced researchers, both mathematicians and physicists, in the area of disordered systems.

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