

Mystic, geometer, and intuitionist. The life of L. E. J. Brouwer. Volume 1: The dawning revolution, by Dirk van Dalen, Clarendon Press, Oxford, 1999, xv+420 pp., £75.00, ISBN 0-19-850297-4

I.

Since his death, the work of Luitzen Egbertus Jan Brouwer (1881-1966) has gained considerable attention. An edition of his *Collected works* appeared in two volumes in the mid 1970s, with some Dutch items welcomely translated into English. His contributions to topology have benefited from historical study, most notably in [4]; and his intuitionism has been studied, especially in various papers and editions by the author (a distinguished logician) and also in [7]. His life has also been of interest; indeed, few great mathematicians of any period have had associated with them such a constellation of stories concerning their personality and behaviour. That life to the early 1920s is recorded in detail in this important study, which concentrates on Brouwer's career and personal (non-)relationships; it also notes some features of his work.

The author draws much from the surviving *Nachlass*, originally formed at his instigation and apparently still at his university in Utrecht (p. vii); he also uses much information obtained from contemporaries and contacts. Where necessary, manuscripts are translated into English, but the original texts are not given, a somewhat worrying point of scholarship. Brouwer's main publications are given in a bibliography, though their locations in the *Works* are not indicated; translations published elsewhere are also not listed. Other primary and historical works are listed separately, but here a significant lacuna emerges. The author refers only twice to the reviewing *Jahrbuch über die Fortschritte der Mathematik* (pp. 130, 161), not even giving proper references to the reviews involved; but much more is to be learnt from that source, as a few examples below will hint.

The text is graced by some illustrations of Brouwer's manuscripts and also by many interesting portraits of colleagues and other photographs; some of the latter have reproduced rather darkly. This is not the only culpability of the publisher: nobody there seems to have copyedited the book before printing, so that the generally excellent English is defaced by some infelicities of language and slips in typing and formatting. The price, however, is top-drawer.

II.

Brouwer's philosophy was partly inspired by an interest in mysticism, which resulted in a short book in 1905. Apparently it was produced quickly (p. 66), and certainly seems so, with original sources quoted but with a poor grasp of their rich and strange background. The bearing of this concern upon his 'intuitionist' philosophy of mathematics lay mainly in two claims: that time was the 'Ur-intuition' of consciousness, and that mathematics was languageless and thus resembled meditation. Part of the mystique around Brouwer is caused by the notorious obscurity (one could say incoherence) of this philosophy. One early and durable consequence

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of its adoption was to drop the law of excluded middle from mathematics, a matter explored in his dissertation of 1907 (ch. 3). Written in Dutch, it is assumed to have been little known until parts were translated into English in the *Works*, but in fact a substantial review was published in the *Jahrbuch* in German by J.A. Barrau, his ‘opponent’ at the viva [2]. Barrau summarised the main features of Brouwer’s position, especially his attitude toward Immanuel Kant and Bertrand Russell on geometry, Cantorian set theory, the ‘logistic’ of Russell and of Giuseppe Peano, and the recent first phase of David Hilbert’s foundational studies. Thus the dissertation may not have been so obscure at the time, although the review does not seem to have inspired published reactions.

Set theory was then gaining general interest after a somewhat controversial earlier development, and it became a particular target for Brouwer. His proofs of 1910-1911 of the Cantor-Bendixson theorem, about decomposing a closed set S into a perfect one and a denumerable one by removing the latter from S to leave a perfect residue, show his early position in a very good light and might have gained some more attention than p. 149: his philosophy was then at an intermediate stage in that he accepted as legitimate the numbers in Cantor’s second number-class but not the class as such (p. 111), and hence re-proof was necessary, since Cantor had used the initial ordinal of the third number-class in his proof. The context for (non-)constructivism in set theory was then rich, mainly because of the controversy over the legitimacy of the axiom(s) of choice, of which a form had been proposed by Ernst Zermelo in 1904; the author notes this link in passing (especially pp. 86, 238), and it has been examined somewhat by others (for example, [5]).

Through consideration of set theory Brouwer gained profound insights about the structure of the continuum and of continuity elsewhere in mathematics. One context was continuous vector distributions, where, in an exceptional but perhaps characteristic move, he published an addendum to two of his own papers in the *Jahrbuch* [1]. A highlight of the mid 1920s was an indecomposability theorem of the continuum; it grew out of his concept of choice sequences to refound the theory of functions (pp. 375-387, including an illustrated manuscript).

Partly in these connections, Brouwer developed his powerful results in topology. Perhaps his best remembered contributions among mathematicians, their merits rest not least in their freedom from intuitionist restrictions! In the early part of the second decade of the twentieth century he came into public conflict with Artur Schoenflies (1853-1928), then Cantor’s chief paladin, by pointing out serious errors. Nevertheless and perhaps unusually on Brouwer’s part, their personal relations seem not to have been disturbed. Sadly, much of the correspondence is lost (p. 210), mainly because Schoenflies’s *Nachlass* was destroyed by the bombing of Frankfurt during the Second World War. Later Brouwer was in touch with Adolf Abraham Fraenkel (1891-1965), another (and better) set theorist. The author records their contact using correspondence obtained from Fraenkel’s widow (pp. 389-401); sadly, its location is now not known after her own death.

A main concern in topology at this time was the invariance of dimension under transformations, a property which Cantor’s mapping of the finite line into the plane on its side had shown to be much trickier than was imagined; efforts to restructure the theory even by Henri Lebesgue had failed. The author describes the struggle in detail in ch. 4, including Brouwer’s opinion of contemporary work: the surmise about the mediocrity of the work of Ludovico Zoratti (p. 165) is confirmed in a letter of 13 February 1913 to Mrs G.C. Young, in the possession of the reviewer,

where Brouwer judged a recent paper to be ‘again full of mistakes.’ The more decisive stage of the 1920s, involving also Karl Menger and Paul Urysohn, will launch the second and concluding volume of this biography.

Among related topics were theorems concerning Riemann surfaces, where Hermann Weyl had contact with Brouwer by means of a *Jahrbuch* review [8] before he came to support a form of intuitionism. Weyl avoided much of Brouwer’s philosophy in his own advocacy (pp. 298-312); the same tendency is evident in Arend Heyting, who completed a dissertation on intuitionist projective geometry in 1925 under Brouwer’s (nominal) direction (pp. 401-404) and then became a major protagonist of intuitionism without the supernatural.

Brouwer also contributed to other mathematical areas. In particular, during and because of the war he worked on photogrammetry (pp. 276-283), publishing some papers which were duly noted in the *Jahrbuch*.

III.

To a degree unusual in a mathematician, Brouwer worked in areas outside his research and teaching. In particular, in the early part of the second decade of the twentieth century he participated in an ‘International Academy of Philosophy’ (pp. 245-250), which re-formed in 1922 as a ‘Signific Circle’ (pp. 367-373). These groups were involved in semiotics, partly under the influence of the English figure Lady Welby. Her reputation is not fully recognised around p. 245; she was a leading follower of the American logician and philosopher C.S. Peirce [3], the modern re-founder of semiotics. Brouwer drew upon the idea of ‘language levels’, which he had already mooted in a form of metamathematics in his dissertation (ch. 10: a parallel with the ‘isotypes’ of the philosopher Otto Neurath is worth noting). He also protested then about the state of university education in occupied Flanders (pp. 275-276).

Brouwer’s private life has stimulated some of the gossip. Unusual in allowing his wife to continue her career as a pharmacist (pp. 53-55), he also roamed around somewhat (pp. 252, 262) and failed entirely to gain the affection of his stepdaughter (p. 374). Among personal disputes, the case of Arnaud Denjoy stands out. Brouwer had played a role in Denjoy’s appointment to Utrecht University in 1917 and membership in the Royal Netherlands Academy three years later (pp. 272, 304); but in 1922 a serious conflict developed, initiated by Denjoy’s submission of a paper to *Mathematische Annalen*, but also involving his attitude toward post-war Germany and maybe his move back to Paris at that time (pp. 344-358).

One source of both some of Brouwer’s personal actions and his difficulties in relationships is an unusually strong and objective sense of justice (marital contexts excluded, presumably). The author brings out this feature well in several cases.

IV.

During the time covered by this book Brouwer was a shining star in the Netherlands. Up to the war he was pretty prolific, and a discussion of his working methods would have been welcome, if evidence were available. Appointed full professor at Amsterdam in 1912, in his early thirties, he gave a lecture on ‘Intuitionism and formalism’. The author notes that Brouwer introduced the name ‘neo-intuitionism’ in this pamphlet (p. 223), but not that ‘formalism’ also received its *début* here. David Hilbert *never* used it to describe his philosophy, which since those times

is regularly misrepresented as claiming that mathematics is just marks on paper. Brouwer's choice of name may have been intended as an inbuilt criticism, but his relationship with Hilbert remained cordial to the extent that Göttingen offered him a chair in 1919, as did Berlin (pp. 300-304).

In the end Brouwer turned down both offers; in the former case this may have been just as well, for during the 1920s his relationship with Hilbert deteriorated so far that in 1929 he was dismissed from the editorial board of *Mathematische Annalen*, which he had joined in 1914 at the invitation of Felix Klein. This affair has caused the most coffee table talk about Brouwer among mathematicians: the author has already described it in some detail in a paper [6], and we can look forward to the full context as one of the treasures of the concluding volume of this biography.

REFERENCES

- [1] Brouwer, L.E.J. 1913. Self-notice of papers 1910A1 and 1910D1 on continuous vector distributions, *Jahrbuch über die Fortschritte der Mathematik*, 41 (1910), 704-705.
- [2] Barrau, J.A. 1910. Review of Brouwer 1907A2, *Jahrbuch über die Fortschritte der Mathematik*, 38 (1907), 81-84.
- [3] Hardwick, C.S. 1977. (Ed. with J. Cook), *Semiotics and signification. The correspondence between Charles S. Peirce and Victoria Lady Welby*, Bloomington (Indiana University Press).
- [4] Johnson, D. 1981. 'The problem of the invariance of dimension in the growth of modern topology', *Archive for history of exact sciences*, 25, 85-267. MR 83e:01050
- [5] Moore, G.H. 1982. *Zermelo's axiom of choice*, New York (Springer). MR 85b:01036
- [6] van Dalen, D. 1990. 'The war of the frogs and the mice, or the crisis of Mathematische Annalen', *Mathematical intelligencer*, 12, no. 4, 17-31. MR 91i:01147
- [7] van Stigt, W.P. 1990. *Brouwer's intuitionism*, Amsterdam (North-Holland). MR 92d:01054
- [8] Weyl, C.H.H. 1915. Review of Brouwer 1912D and 1912G, *Jahrbuch über die Fortschritte der Mathematik*, 43(1912), 527-528.

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