BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 37, Number 1, Pages 106–107
S 0273-0979(99)00833-2
Article electronically published on December 21, 1999

Foundations of algebraic geometry, by André Weil, American Mathematical Society Colloquium Publications, vol. 29, American Mathematical Society, New York, 1946, 20 + 288 pp., $5.50

In the words of the author the main purpose of this book is "to present a detailed and connected treatment of the properties of intersection multiplicities, which is to include all that is necessary and sufficient to legitimize the use made of these multiplicities in classical algebraic geometry, especially of the Italian school." There can be no doubt whatsoever that this purpose has been fully achieved by Weil. After a long and careful preparation (Chaps. I–IV) he develops in two central chapters (V and VI) an intersection theory which for completeness and generality leaves little to be desired. It goes far beyond the previous treatments of this foundational topic by Severi and van der Waerden and is presented with that absolute rigor to which we are becoming accustomed in algebraic geometry. In harmony with its title the book is entirely self-contained and the subject matter is developed *ab initio*.

It is a remarkable feature of the book that—with one exception (Chap. III)—no use is made of the higher methods of modern algebra. The author has made up his mind not to assume or use modern algebra "beyond the simplest facts about abstract fields and their extensions and the bare rudiments of the theory of ideals." Weil's faithful realization of this program of strict mathematical economy is an achievement in itself. In some cases this leads to the "best possible" proofs. However, on the whole one may question the wisdom of this self-imposed régime of austerity. The methodical reduction of the theory—which is both difficult and subtle—to the primitives of algebra is bound to be a very laborious process. As a result, the reader finds himself very much in the position of a man who must collect a large amount of cash most of which is in pennies. The author justifies his procedure by an argument of historical continuity, urging a return "to the palaces which are ours by birthright." But it is very unlikely that our predecessors will recognize in Weil's book their own familiar edifice, however improved and completed. If the traditional geometer were invited to choose between "makeshift constructions full of rings, ideals and valuations" on one hand, and constructions full of fields, linearly disjoint fields, regular extensions, independent extensions, generic specializations, finite specializations and specializations of specializations on the other, he most probably would decline the choice and say: "A plague on both your houses!" That being so, we may just as well help ourselves to modern algebra to the fullest possible extent.

. . .

Weil's book is the first purely arithmetic exposition of an important sector of algebraic geometry, and is therefore a landmark in the literature of this field. This, and the competence of the author, give the book added significance.

In the remainder of the book we wish to recommend especially the interesting chapter entitled *Comments and discussions*, where various unsolved problems are

Reprinted from Bull. Amer. Math. Soc. **54** (1948), 671–675.

discussed and possible directions of future research are indicated. The book has an excellent list of definitions and table of notations.

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