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Selected papers on the classification of varieties and moduli spaces, by David Mumford, Springer-Verlag, New York, 2004, xiv+795 pp., US$99.00, ISBN 0-387-21092-X

It was quite a surprise when the request to review the selected works of David Mumford came, since I do not know him at all. By the time I got to Harvard, his interest had already turned from algebraic geometry to vision. During my three years there we never spoke, and I saw him rarely, if at all. I may be the first of the generation to whom Mumford is not a person, but rather a legend who exists only through his writings. There are many people who know him well, so why not ask them to write a review? On the other hand, in this case, the reviewer's job is precisely to comment on the published record of Mumford, so it may be, after all, an appropriate task for someone who has only his writings to rely on. For those who wish to read a more personal account, the hagiography of Mumford still waits to be written about a mathematician who, in the view of many, was capable of "superhuman effort" [Mum66, p. 288].

The 60's must have been a great time to be an algebraic geometer. Grothendieck and his group were rewriting the whole subject in Paris, building on the theory of sheaves that had been recently brought to algebraic geometry by Serre. Kodaira, Weil and Zariski represented the best of the older schools in the USA, and a new generation suddenly turned a small field into one of the major branches of mathematics. Starting as students in the 60's, Artin, Clemens, Fulton, Griffiths, Hartshorne, Hironaka, Katz, Kleiman, Mumford and many others turned the 70's into the golden age of American algebraic geometry. When, around 1980, some of the leaders of this generation left the field, it felt like the "extinction of the dinosaurs" [Rei87, p. 333].

The dominant personality of the 60's was Grothendieck. Several American algebraic geometers worked on various parts of the monumental volumes of [Gro69]. By contrast, Mumford established his own brand of algebraic geometry. Before we examine what this "Mumfordian" algebraic geometry was, a brief historical tour is in order.

The great Italian school of Castelnuovo, Enriques and Severi was a group of *geometers*. Their background and all their instincts came from the classical geometry of 2 and 3 dimensions and from function theory. By the 1920's mere geometric intuition started to be insufficient, and in the axiomatic era started by Hilbert, the Italian methods did not stand up well. They still proved great results, but the proofs lacked rigor, and with disconcerting regularity some of their "theorems" were actually wrong. The first wave of putting algebraic geometry on solid, meaning algebraic, foundations was accomplished by van der Waerden, Weil and Zariski. By 1950 the subject turned into *algebraic* geometry. In the process, however, much of the accumulated work of the past 100 years was lost. Then came the revolution of Grothendieck, who elevated the subject to a level of abstraction never seen or even imagined before. Many felt that he turned algebraic geometry into a chapter of category theory. Had they been able to meet, Castelnuovo and Grothendieck could

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have spent quite a lot of time talking without realizing that they were considered masters of the same subject.

Faced with this widening gulf, Mumford's work over twenty years did much to join together the methods of Grothendieck with the ideas and questions of the Italian school. While the 60's and early 70's were dominated by the Grothendieck school, the next two decades saw the return of classical geometry. The abstract methods were applied to questions that were raised 100 years earlier. Castelnuovo would have felt quite at home with the main theorems of algebraic geometry in the 80's.

Historians have long debated whether great men turn the tide of history or they are simply the first to see that the tide is turning and get in front of it. Anyway one thinks about it, Mumford (relying on the machinery of Grothendieck) and Griffiths (using complex analytic techniques) had the greatest influence during this change.

This brings us, finally, to the volume under review. Is it a good summary of David Mumford's work in algebraic geometry?

It is only "selected works", and one can certainly quibble with the selection. I very much like his first paper, "The topology of normal singularities of an algebraic surface and a criterion for simplicity" [Mum61], but it clearly does not fit into any of the three main themes around which the volume is organized. His paper "Picard groups of moduli problems" [Mum65b] is more important historically, since this is one of the first places where the "stacky" viewpoint of moduli problems clearly appears. Though Mumford himself did not follow up on it much, stacks have gained in importance considerably in the last decade. It is even less comprehensible why "The irreducibility of the space of curves of given genus" [DM69] is missing from the collection. Among Mumford's papers it has by far the highest number of reference citations on MathSciNet, and the study of the moduli of curves is probably the subject most closely associated with Mumford. Its absence is a real loss. (Is it, perhaps, waiting for the collected works of Deligne?)

In order to gain a true understanding of Mumford's contributions to pure mathematics, one should also study his books. Unlike most mathematicians, Mumford published several of his major works only in book form. The most famous among them is *Geometric Invariant Theory* [Mum65a], known as GIT. Together with Grothendieck's EGA and SGA [Gro67, Gro69], these are the only books in the subject generally known by their initials. GIT is the work that established Mumford's reputation and to a large extent earned him the Fields Medal [Tat75]. One can get some idea of GIT from the papers in this volume, but anyone who intends to study Mumford's legacy should start with GIT. Similarly, *Toroidal Embeddings* [KKMSD73] and *Smooth Compactification of Locally Symmetric Varieties* [AMRT75] are essentially extended research papers masquerading as books.

What is, then, the point of publishing Mumford's selected works in a volume? Or for that matter, anyone's selected or complete works? These days many of the old journals are accessible on the Web, and almost all of Mumford's papers are easy to find in mathematical libraries. (A few of his works appeared in less well-known journals and proceedings, but these are also missing from the current selection.)

While I cannot answer the general question, at least I can say that having perused this volume in the last few months, I now view the heritage of Mumford in a different light. Beforehand, I considered GIT to be his main work, and I believe that GIT did not fulfill all the early expectations. The first successes leading to the construction of the moduli spaces of curves, Abelian varieties, vector bundles and sheaves were not carried much further. For instance, GIT never came up with a good approach to compactify the moduli space of surfaces of general type, and it did not tackle the moduli problem for higher dimensional varieties.

Now I see GIT as but one part of the total effort, which turned the focus of algebraic geometry back to classical questions. It is, for instance, a real pleasure to read the series of short gems "Pathologies I–IV" and to contemplate how much they changed our view of what to expect in algebraic geometry. So I am quite happy to keep this volume on my shelf, and I will surely find many more seeds in it that grew so large that by now their origins are hard to recognize.

The volume under review divides Mumford's papers into three broad areas, each preceded by an essay summarizing the results and outlining their influence on further developments. These commentaries are: D. Gieseker on moduli problems, the late G. Kempf and H. Lange on Abelian varieties, and E. Viehweg on the classification of surfaces. Mumford worked in many different areas of algebraic geometry, and one can plausibly claim that any thorough survey of the influence of Mumford's work should encompass all contemporary algebraic geometry and some related fields as well. The three introductions are necessarily more limited in scope. They stress the earlier works which were directly influenced by Mumford, but are rather sketchy when it comes to describing the work of the last decade.

I have only some minor observations and corrections. I suspect from the context on p. 7 that [M2] on p. 17 should have been "'On the Kodaira dimension of the moduli space of curves" [HM82] rather than "On the Kodaira dimension of the Siegel modular variety" [Mum83]. Also, two misspelled names are Iarrobino (on p. 6) and Caporaso (on p. 10).

I find references such as "papers too numerous to mention" on p. 293 and "papers. . . too many to be quoted here" on p. 296 rather unhelpful.

On p. 651, in connection with Thm. 6, the newer results of Chen–Hacon [CH01] could have been mentioned, and the discussion of the Lüroth problem on p. 653 would have been helped by some recent references, for instance the volume [CR00], which contains both survey and research papers.

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