MATHEMATICAL PERSPECTIVES

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 43, Number 4, October 2006, Pages 563–565
S 0273-0979(06)01127-X
Article electronically published on July 18, 2006

MATHEMATICIANS AND PHYSICISTS = CATS AND DOGS?

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An affirmative answer to this question was given in a referee's report written by a famous physicist about a paper written by a famous mathematician.

The text below closely follows my talk during the round-table discussion at the 95th Conference on Statistical Physics at Rutgers, May 2006, organized by J. Lebowitz. The subject of the discussion was "The Unreasonable Effectiveness of Mathematics in Natural Sciences", formulated by Eugene Wigner in his Courant Lecture at New York University. Wigner's lecture was delivered on May 11, 1959, i.e. almost 50 years ago. Other participants in this recent discussion were Ph. Anderson, F. Dyson, and E. Witten. The chairman of the session was M. Fisher. Wigner published a paper with the same title in CPAM, vol. 13, No. 1. It starts with the following story. A young statistician who was working on problems of the growth of population explained to his friend the difficulties which he had encountered and showed him some results of his analysis. The friend noticed π in his formulas and asked what it meant. The statistician replied that π is the area of the circle having radius 1. Then the friend said, "Do you want to convince me that the area of the circle has something in common with the growth of population?"

At the end of his paper Wigner also wrote: "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend for better or for worse to our pleasure even though perhaps also to our bafflement with the wide branches of learning."

The famous papers by Wigner on ensembles of random matrices appeared several years before that paper. It is hard to imagine modern theoretical physics without algebraic geometry and topology. On the other hand, theoretical physics, especially string theory, provides a lot of beautiful and important problems to this part of mathematics. However, physicists did not always have a great appreciation

Received by the editors May 23, 2006.

of mathematics. The leading Russian physicist L. Landau once said that the best physicist in Russia was Ya. Frenkel, who used in his papers only quadratic equations. Landau himself was slightly worse, because sometimes he needed ordinary differential equations. The mathematical part of the famous Landau theoretical minimum contained only problems on integration, vector calculus, and ODE. Remarks in the same style about mathematics are scattered throughout the textbooks by R. Feynman. Mathematicians responded to this by saying that physicists dealt with mathematics as criminals do with criminal codes (I. M. Gelfand). Some reasons for this animosity were obvious. In mathematics the dominant style was based on an axiomatic approach, epsilon-delta proofs, and strong requirements for rigor. In physics there was a dominant trend toward complicated schemes of perturbation theory, diagrams, etc., which were very difficult for classical mathematicians to appreciate. Clearly, this was a cold "cat-dog" era.

Later, mathematicians started to regularly attend seminars and conferences in physics, and the number of mathematicians who understood deeply the problems of physics increased significantly during the lifetime of one or two generations. It seems that this process started at the end of the fifties when physicists realized that they could find something useful in modern mathematics that they did not know. Let me give two examples connected with the famous "KAM-theory".[1] One physicist told me that KAM-theory was so natural that it must have been invented by physicists. At the end of the fifties two famous Russian physicists, L. A. Arzimovich and M. A. Leontovich, came to the Moscow State University seminar led by A. N. Kolmogorov to explain the problem of existence of magnetic surfaces, which was of great importance at that time. L. A. Arzimovich, an experimentalist, was the speaker. His talk was very clear and inspiring, and shortly after that V. Arnold solved the basic problem by using KAM-theory. This event can be considered as the end of the cold cat-dog era.

In my opinion, it is very important and useful for mathematicians who decide to work on problems related to physics to have more-or-less regular contacts with physicists. I used to meet quite often with I. M. Lifshitz, who was a leading theoretical physicist of his time. When we met for the first time he asked me what I was doing. When I answered that it was ergodic theory, he remarked, "Ergodic theory is a theory which explains that every shoelace sooner or later becomes disentangled." During my next visit I tried to explain to Lifshitz the joint results with my student S. Pirogov on phase diagrams of lattice models at low temperatures. He started to listen but then very quickly said that everything was very simple and obvious. Then he wrote several formulas deriving our results. I left very much embarrassed, and only after some time did I realize that the formulas for the logarithms of partition functions that he used were the final and most difficult result of our theory.

A similar reaction came from I. M. Gelfand. When we explained the results to him, he remarked that for physicists everything should be obvious. However, when we asked him whether we should write a text with the detailed exposition of the whole theory, he answered, "Sure, yes."

There were many other cases when the reactions of physicists were surprising and very much different from the ones of mathematicians. Once, I returned to Moscow from my trip to the USA and explained to my friend who was a physicist the hypothesis which I had heard from T. Spencer about the abundance of the values

[1]KAM = Kolmogorov-Arnold-Moser

of the parameter for which the standard map had no KAM-islands. My friend thought for a while and then said, "It must be a great mathematical theorem, because we physicists never saw this." However, sometime later, he wrote in one of his subsequent publications that, as everybody knows, the standard map has values of parameters for which there are no islands.

Sometimes mathematicians understand too literally the statements or results by physicists. Several years ago there was a paper by M. Berry and M. Tabor in which they claimed that the distributions of spacings between the nearest eigenvalues of Laplacians of integrable metrics converge to the Poissonian law. From a probabilistic point of view the statement looks very appealing, and I spent a couple of years trying to prove it. Finally, I could show that it is true for random integrable metrics. As far as I know nothing yet is proven for concrete metrics. I recently discussed the issue with a physicist, and he told me that they understand, under Poissonian law, the statement that the second correlation function tends to a positive limit as the distance tends to zero. Clearly, this implies the absence of repulsion of levels, which is the main issue. But this is a much simpler theorem which can be easily proven under very general conditions.

Now it is more or less usual to invite mathematicians to give talks at physics seminars. After one such talk at a big seminar on theoretical physics, the chairman asked me about possible applications of my results to experimental physics. I replied that for me theoretical physics played the same role as experimental physics played for him. It was not a joke. Usually I do not trust physicists until I find my own proof or, at least, an explanation of their results. For this reason, a big part of theoretical physics remains outside my understanding. My late friend R. L. Dobrushin once remarked that every mathematician builds for himself his own theoretical physics. This is certainly an exaggeration. However, it is true that the worlds of mathematicians and physicists are quite different and there is a boundary which separates them. This boundary is very individual, and everybody chooses it for himself.

About the author

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