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ABOUT THE COVER: KUMMER'S TABLES

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The meticulously drafted table on the cover gives a startling glimpse of Ernst Eduard Kummer's method of work. Although anyone who reads Kummer's papers can see that he thought algorithmically and that his discoveries were based on the experience of extensive calculations, this table reveals that the scope of his calculations was beyond what anyone today would think was feasible without computing machines.

The story behind the tables is told in a communication Kummer made to the Berlin Academy in 1850 [3], in which he explains that his work in 1846–7 on the arithmetic of cyclotomic integers had led him to conjecture a certain reciprocity law for this arithmetic. (He doesn't use the modern term "cyclotomic integers" and calls them "complex numbers", but he was dealing with a very special integral domain contained within the field of complex numbers, namely, for a fixed prime λ , the integral domain generated over the integers by a primitive λ th root of unity.) The exact statement of his conjecture is not difficult to give, as the interested reader can see by reading his statement of it in the paper. (For an explanation in English, see [1].) But the subject here is not the law itself but the extensive tables Kummer compiled to convince himself of its truth.

The tables were enough to convince him that his conjecture was correct, or at least to be convinced enough to want to tell his mentor and cousin by marriage, G. Lejeune Dirichlet, about it. The tables have come down to us because they accompanied the resulting letter of January 20, 1848, to Dirichlet and because Dirichlet kept them. (One would assume Kummer, being a prudent man, kept copies for himself, but few of Kummer's papers have survived, and as far as I know no other copies of the tables survive.) Kummer was a professor in Breslau at the time, and Dirichlet was a professor in Berlin, where he was in constant communication with C. G. J. Jacobi. Kummer gave Dirichlet permission to share his letter with Jacobi, but asked that it not go farther because, naturally, he was

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hopeful he would be able to prove his conjecture and didn't want to give anyone the chance to beat him to it.

But two years passed without his finding a proof, and he decided, as he said, to make this discovery “the common property of all mathematicians working to advance number theory” by publishing it in the proceedings of the Berlin Academy in May of 1850. I find this altruistic reason for publishing the conjecture both admirable and credible, but I think it is not irrelevant that Kummer's talented and hard-working rival, G. Eisenstein, was working on very similar ideas and that Kummer knew, through Dirichlet and Jacobi, that Eisenstein would soon be publishing something on the subject.

(As it turned out, Eisenstein's contributions to the subject, although they were very substantial, approached the problem in a different way and were not directly competitive with Kummer's. Eisenstein's early death at 29 in 1852 ended the competition without there being a winner or a loser, and Kummer ultimately did prove that his law was correct, but not until almost a decade later. The proof [4] was published in 1859. For more on the Kummer-Eisenstein rivalry, see [1].)

Kummer's computational methods and the mathematical culture out of which they came are illuminated by his reference to Jacobi's *Canon Arithmeticus*. This remarkable book, published in 1839, contains tables of indices for all primes less than 1000—that is, it contains the analog of tables of logarithms for the multiplication mod p of numbers not divisible by p . Kummer was able to use them to test his reciprocity law in a large number of cases for small values of λ for which the integral domain is a principle ideal domain, but he says (p. 160 in the original pagination) that he stopped at a point at which “the *Canon Arithmeticus* can no longer be successfully applied and the computations become too time-consuming.” One yearns to ask him how time-consuming the computations had been up to that point! But his Table IV deals with cases in which one of the two numbers in the reciprocity formula is ideal, and the *Canon Arithmeticus* does not seem to have played a role in these verifications of his law. As Kummer points out, the table verifies 946 cases; he does not mention that each case requires the evaluation of 2 “Legendre symbols” to obtain the 1892 entries of Table IV. It would be my guess that he did these computations single-handedly.

In 1974–5, when the *Collected Papers* of Kummer were being published by Springer-Verlag under the editorship of André Weil, I suggested to Walter Kaufmann-Buhler of Springer that one of Kummer's tables be used as a frontispiece for one of the volumes. Kaufmann-Buhler later told me that Weil had rejected the idea. I never heard Weil's response first-hand, but Kaufmann-Buhler told me Weil had said “they all did that sort of thing then,” which I took to mean that Weil thought this was not a remarkable aspect of Kummer's work or even that he thought it showed a weakness on Kummer's part that needed to be overlooked. I believe, on the contrary, that even though many of Kummer's contemporaries may have done similar things, the tables show an important aspect of his mathematical work and show the true foundation of his great achievements.

(The table is reproduced with the permission of the manuscripts section of the Preussischer Kulturbesitz, Staatsbibliothek zu Berlin.)

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