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James Joseph Sylvester: Jewish mathematician in a Victorian world, by Karen Hunger Parshall, The Johns Hopkins Univ. Press, Baltimore, 2006, xiv+461 pp., US$69.95, ISBN 0-8018-8291-5

This is one of the rare books - Joseph Dauben's *Georg Cantor* [2] and Tony Crilly's *Arthur Cayley* [1] are the most obvious comparisons - that both put a mathematician's life into full social context and do justice to his mathematics. One index to the importance of James Joseph Sylvester (1814-1897) is the fact that he coined the common algebraic terms *matrix* and *invariant*, together with many others, including annihilator, canonical form, commutant, covariant, discriminant, Hessian, Jacobian, minor, nullity, umbral, and weight. Also, his own name has been given to results in both algebra and in number theory called "Sylvester's theorem", together with the Sylvester matrix, Sylvester's inertia law, the Sylvester cyclotomic number, Sylvester's signature, Sylvester's determinant identity, the Sylvester graph, Sylvester's inertia law, Sylvester's sequence, the Sylvester 4-point problem, the Sylvester-Gallai theorem, Sylvester's criterion for a positive definite matrix, and the Chebyshev-Sylvester constant.

But Sylvester's interest for modern mathematicians goes far beyond his mathematics. His mercurial personality and colorful life are fascinating and tell us a lot about what it meant to be a mathematician in the English-speaking world in the nineteenth century. Sylvester was Jewish - not observant, but also not willing to deny his faith - when many positions in England were still legally closed to professing Jews. In fact, although Sylvester finished second in the Cambridge Mathematical Tripos in 1837, he could not even officially receive his Cambridge degree until 1871. He aspired to combine teaching and research to be a professional mathematician in his home country, but since he could not swear that he was a Christian, Oxford and Cambridge were closed to him. Sylvester bounced from job to job and even crossed the Atlantic twice. He was briefly a professor of natural philosophy at University College, London: served a briefer, disastrous stint at the University of Virginia; worked as an actuary and qualified as a lawyer in London; taught mathematics at the Royal Military Academy at Woolwich; was forced to retire; and, finally, got a job befitting a professional mathematician when he was hired by Daniel Coit Gilman, the visionary president of a new American university, Johns Hopkins, in 1876. Nabbing someone of Sylvester's eminence for Johns Hopkins was not the only time America has profited from religious restrictions in Europe. While there, Sylvester kick-started the nascent American mathematical research community and was instrumental in founding the *American Journal of Mathematics*. After the religious restrictions in England were finally repealed, Sylvester chose to return home after the death of H. J. S. Smith in 1883 to accept the Savilian Chair at Oxford.

The outline of this story has sometimes been given, but it has never been told in its full and rich context. Karen Hunger Parshall is the person to do it, and she has done it superbly in the book under review. Her knowledge of Sylvester

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is unequalled. This fact is obvious to any reader of her fine earlier work, *James Joseph Sylvester: Life and Work in Letters* [4], in which she chose and reproduced 140 letters of the more than 1,200 that exist. *Life and Work in Letters* includes letters between Sylvester and many famous nineteenth-century mathematicians and scientists, including Michel Chasles, Augustus De Morgan, Sir Francis Galton, Paul Gordan, Joseph Henry, Charles Hermite, Sir John Herschel, Camille Jordan, Felix Klein, Charles Sanders Peirce, George Salmon, William Spottiswoode, and so many letters to and from Arthur Cayley that he and Sylvester might as well have been corresponding by email. The letters are chronologically arranged, and for each period of Sylvester's career, Parshall provides a brief introductory section summarizing the key events. More importantly, she provides insightful explanatory footnotes, both mathematical and historical, whose volume surpasses that of the text of the letters. This earlier book is a fascinating read. Since the notes explain almost everything that a modern mathematical reader might need explained, the letters let us follow Sylvester's thinking throughout his research, let us watch Sylvester and Cayley bounce ideas off each other: Parshall says "Sylvester was a natural extrovert as a mathematician; he thrived on mathematical interchange" [3, p. 95]. The letters also help us understand Sylvester's role in the American mathematical community while he was at Hopkins. Anyone interested in Sylvester's work would enjoy reading *Life and Work in Letters*.

The Sylvester correspondence that has survived, however, is principally about mathematics. Even though Parshall's collection includes such items as Sylvester's unsuccessful proposal of marriage to the British feminist Barbara Leigh Smith; the letter from Sylvester, Simon Newcomb, Henry Rowland, and William Story announcing the plan for founding the *American Journal of Mathematics*; and Sylvester's 1883 letter thanking Cayley for telegraphing him about "the gratifying and unexpected intelligence of my election to the Savilian Professorship" [4, p. 229], there is a lot missing from the letters. In particular, we learn nothing about Sylvester's youth and early ambitions, how he got involved with actuarial work and what he did as an actuary and lawyer, or how things went when he tried to teach serious mathematics to young would-be military officers at Woolwich. Nor, in these letters, did Sylvester address his Jewish heritage. Anybody wanting a full reconstruction of Sylvester's life and world is left asking for more and wondering where, if not in the letters, that more could be found.

The answer lies in the volume under review. Having found and analyzed all sorts of sources – manuscripts, correspondence, university catalogues, rules for actuaries, articles of incorporation of various institutions, documents from the history of the Jewish communities of London and Liverpool, newspapers, biographies of other figures - Parshall gives us a three-dimensional picture of Sylvester the mathematician in full social and historical context. The reader feels immersed in Sylvester's world. One learns much about nineteenth-century Britain, the international mathematical community, the birth of research mathematics in the United States, what it must have been like to be a Jewish mathematician - or any other sort of outsider – in Victorian England, and, of course, the history of algebra. Interestingly, Parshall says in the Preface that she came to focus fully on Sylvester's Jewish identity when Professor Dan Silver invited her to give a mathematics colloquium at the University of South Alabama, because he also arranged for her to speak about Sylvester to the Jewish community of greater Mobile. The religious openness and diversity of America thus helped shape not only Sylvester's career but also Parshall's biography.

Sylvester's overall importance falls under two headings: the mathematical and the social. Sylvester and Cayley invented English invariant theory. Cayley in 1845 defined the chief task of invariant theory as "to find all the derivatives of any number of functions, which have the property of preserving their form unaltered after any linear transformation of the variables" [3, p. 364, n. 101]. Sylvester in particular began with homogeneous polynomials in any number of variables, exploring their structure - what he called their "morphology". Like a nineteenth-century British naturalist who wanted to name, classify, and describe the plant and animal kingdoms, Sylvester wanted to name, classify, and describe the invariants in the mathematical world. Sylvester also wrote on Sturm's function, elimination, the partitions of numbers, and matrix theory, as well as proving Newton's Rule for finding the complex roots of polynomial equations. Max Noether praised Sylvester's "truly combinatorial-algebraic genius." The Sylvester-Cayley invariant theory was important and essential, but it was eventually superseded by the work of Aronhold, Clebsch, Gordan, and, above all, David Hilbert, who saw himself shifting the emphasis, as Parshall puts it, "from the exhibition of and search for concrete examples in successively higher degrees and numbers of variables to the development of sweeping existence theorems that handled all of the specific examples in one fell swoop" [3, p. 334].

Sylvester also left a social legacy. As an outsider in English society, he sought a model for a career as a professional man, where competence, not social background, mattered and where his work would secure him both recognition and a decent living. The ideal he wanted to pursue was to combine teaching his subject, doing research at a high level, and training future researchers. And at Johns Hopkins, finally, he was able to do this. As one of his students there, George Bruce Halsted, put it, "All young men who felt within themselves the divine longing of creative power hastened to Baltimore, made at once by this Euclid a new Alexandria" [3, p. 1]. He set up the first real mathematics research program in the United States, an example, based in part on the German model, soon followed by Chicago, Cornell, Harvard, and Yale. Although when Sylvester returned to England as Savilian Professor at Oxford he was unable to establish such a role there - mathematics was even in 1880s Oxford seen as a way to teach mental discipline, and Oxford was still in transition from a set of church-oriented colleges to becoming a modern university – his example "helped to shape a new professional category" [3, p. 335]. He was an example also in being the first Jewish professor at Oxford or Cambridge.

There are fascinating facts in this book, many of which are new. One learns that, after his unsuccessful job at the University of Virginia in 1843, he tried to get jobs at Harvard and at Columbia. In the latter case, although Columbia, founded by Episcopalians to be sure, was officially nonsectarian, the selection committee told him that choosing a Jew "would be repugnant to the feelings of every member of the board," even if he had been born in the United States [3, p. 80]. What had gone wrong at Virginia seems to have been due to rowdy and undisciplined students, not to mention xenophobia and religious prejudice, and a contemporary reported that Sylvester was not the only faculty member who bought a sword cane to defend himself [3, p. 75]. Also, when trying to get the job at the Royal Military Academy, Sylvester enlisted Charles Hermite to get him a letter of support from Dirichlet; Hermite pointed out to Dirichlet how poorly mid-nineteenth-century

England, as opposed to France or Germany, supported its scientists. Hermite apparently thought that the Military Academy was like the École Polytechnique. This was far from true; the students found Sylvester too theoretical. Again, when Thomas Henry Huxley, the great apostle of Darwinism, attacked dogmatic teaching in the name of observational science, mathematics suffered some collateral damage: Huxley said in 1869 that mathematics, unlike the rest of science, "knows nothing of observation, nothing of experiment, nothing of induction [meaning generalization from observations], nothing of causation" [3, p. 204]. Sylvester, in his presidential address to Section A of the British Association for the Advancement of Science, struck back, saying that mathematics "unceasingly calls for the faculties of observation and comparison," induction was one of its principal weapons, and above all "it affords a boundless scope for the exercise of the highest efforts of imagination and invention." Sylvester did agree, though, that science education needed to be reformed, and in mathematics this meant "Euclid honourably shelved or buried 'deeper than did ever plummet sound'" being replaced by "mathematics taught with...life and animation" [3, p. 205].

The most exciting part of this book for this reviewer is the part where we follow Sylvester's wonderful seven years at Hopkins. Sylvester wrote a friend, "It is impossible to imagine a more generous and appreciative mode of treatment by any institution of its professors" [3, p. 231]. President Gilman hosted a welcoming dinner in Sylvester's honor in November of 1876, inviting Sylvester's scientific friends Joseph Henry and Benjamin Peirce, but also the astronomer Simon Newcomb and the locals Henry Rowland and Joseph Story. Gilman's getting these men together resulted in the founding of the *American Journal of Mathematics*. Compared with the Royal Military Academy, writes Parshall, "Johns Hopkins was a scholar's paradise." "Never before had [Sylvester] taught students who wanted to be in his classroom simply for the sake of learning...been given free rein to pursue his own researches as part of his academic mission...had his teaching and his research been mutually reinforcing" [3, p. 234]. His students, working with him, "found themselves in a kind of mathematical laboratory in which experiments...sometimes succeeded and sometimes failed, but in which new mathematics was being created" [3, p. 236]; they experienced "the satisfaction of the successful mathematical hunt" and "the emotion that could accompany the research enterprise." Sylvester also found Hopkins remarkable for its nonsectarianism, by contrast with English universities, which, Sylvester said, "until so late a period [had been] the monopoly of a party and the appanage [sic] of a sect" [3, p. 235]. In keeping with his ideals, Sylvester strongly championed the application of Christine Ladd, a Vassar graduate of 1869, to become a graduate student at Hopkins. He lost this battle; the trustees ultimately decided not to make the institution coeducational, though they allowed her to study with Sylvester unofficially. Finally, when Sylvester decided to leave Hopkins for Oxford, the entire Hopkins university community turned out for a farewell reception in his honor.

Sylvester was, as Parshall says, not among the greatest mathematicians of his century. Indeed, his friend Arthur Cayley surpassed him. Still, Professor Parshall has written an exemplary biography of a fascinating mathematician, illuminating both his career and the nineteenth-century development of mathematics as a profession. I highly recommend the earlier *Life and Work in Letters* and, even more,

the present volume, the definitive account of the life of James Joseph Sylvester and his world.

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